Systems of equations: simultaneous equations

Lesson plan and teacher's notes · Form 2 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 2
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Systems of equations: simultaneous equations

Learners write equations for shop and market situations, solve one-variable equations with the balance idea, and solve simultaneous equations by elimination, checking and stating the answer in words. The lesson uses prices of exercise books, pens, tomatoes and onions.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Explain what a variable, an equation and a system of equations are
2. Write one or two equations for a real situation
3. Solve a one-variable equation using the balance idea and check it
4. Solve simultaneous equations by elimination, check in both equations and state the answer in words

Prior knowledge

- Multiplying and dividing whole numbers and decimals
- Adding and subtracting positive and negative numbers
- Using a letter to stand for an unknown number

Teaching and learning materials

- chalk and board
- two sets of price cards for items at a school shop
- exercise books

References

Ministry of Education (CDC) Form 2 Mathematics syllabus and module; 25Schools learners' note: Systems of equations: simultaneous equations.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Say that two friends bought items at the school shop and paid different totals. Ask how we could find the price of one item. Write variable and equation on the board.	Suggest ways to find one price and say what is unknown.
Development: unknowns and equations 10 min	Work Situation A: three exercise books and a ruler of K6 cost K39, so 3x + 6 = 39. Solve by the balance idea and check. Then introduce Situation B with two unknowns.	Solve 3x + 6 = 39 step by step and check by substituting.
Development: elimination 15 min	Teach the five steps of elimination using 2x + y = 23 and x + 2y = 19. Multiply, add or subtract, solve, substitute, check. Show when to add and when to subtract.	Copy the steps and solve the system with the teacher, then repeat on x + y = 10 and x - y = 4.
Practice 7 min	Set the tomatoes and onions problem. Move round the class and check that each learner writes the answer in words.	Solve in pairs, check in both equations and state the prices.
Conclusion 3 min	Ask two learners to say the five steps of elimination in their own words. Set the homework.	Answer and copy the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

Variable and equation

A variable is a symbol such as x or y that stands for an unknown quantity. An equation says two expressions have the same value. A linear equation has the form ax + b = c, or ax + by = c with two variables.

System of equations

Two or more equations considered together form a system. In simultaneous equations the same unknowns must satisfy both equations at the same time. The solution is the pair of values that fits both.

Balance idea

For 3x + 6 = 39, subtract 6 from both sides to get 3x = 33, then divide both sides by 3 to get x = 11. Always check by substituting back: 3(11) + 6 = 39.

Elimination

Make the coefficient of one variable the same in both equations by multiplying. Subtract the equations if the coefficients have the same sign and add them if they are opposites. Solve for one variable, substitute into an original equation, then check in both.

Stating the answer

State the answer in words in the situation, such as an exercise book costs K9 and a pen costs K5. Modules also name substitution and the graphical method; this lesson uses elimination.

Questions to ask during the lesson

Question	Expected answer
A learner buys three identical exercise books and a ruler that costs K6, and the total is K39. (a) Write the equation using x for the price of one exercise book. (b) Solve it. (c) Check your answer.	(a) 3x + 6 = 39. (b) 3x = 33, so x = 11. (c) 3(11) + 6 = 39, which is correct.
A teacher writes x + y = 10 and x - y = 4 on the board and asks the class to find the two numbers. (a) Say whether to add or subtract and why. (b) Find x and y. (c) Check in both equations.	(a) Add, because y has opposite signs. (b) 2x = 14, so x = 7 and y = 3. (c) 7 + 3 = 10 and 7 - 3 = 4, both correct.
A seller at a market near Feira gets K30 from one customer for 2 kg of maize and 1 kg of beans, and K35 from another for 1 kg of maize and 3 kg of beans. (a) Write two equations with m and b. (b) Say how to eliminate m. (c) Find the price of each.	(a) 2m + b = 30 and m + 3b = 35. (b) Multiply the second by 2 to get 2m + 6b = 70 and subtract the first. (c) 5b = 40, so b = 8 and m = 11. Maize costs K11 and beans cost K8 per kilogram.
A learner solves two equations and finds x = 6 and y = 7 but only checks in the first equation. (a) Say what is missing. (b) Say why it matters. (c) Do the check for 2x + y = 19 and 2x - y = 5.	(a) A check in the second equation. (b) The solution must satisfy both at the same time. (c) 2(6) + 7 = 19 and 2(6) - 7 = 5, both correct.

Worked examples for the board

Example 1: tomatoes and onions

2 kg of tomatoes and 3 kg of onions cost K42, and 3 kg of tomatoes and 2 kg of onions cost K43. Let x be tomatoes and y onions: 2x + 3y = 42 and 3x + 2y = 43. Multiply the first by 3 and the second by 2 to get 6x + 9y = 126 and 6x + 4y = 86. Subtract: 5y = 40, so y = 8. Then 2x + 24 = 42, so x = 9. Tomatoes cost K9 per kilogram and onions K8.

Example 2: add to eliminate

Solve 2x + y = 19 and 2x - y = 5. The y terms have opposite signs, so add: 4x = 24, so x = 6. Then 12 + y = 19, so y = 7. Check in 2x - y = 5: 12 - 7 = 5, which is correct.

Example 3: the school shop

2x + y = 23 and x + 2y = 19. Multiply the second by 2: 2x + 4y = 38. Subtract the first: 3y = 15, so y = 5. Then 2x + 5 = 23, so x = 9. Check: 9 + 2(5) = 19. An exercise book costs K9 and a pen costs K5.

Board plan

Write the title. Left: variable, equation and system with one example each. Centre: the five steps of elimination in a table, with the school shop example worked below. Right: the check in both equations and the answer sentence.

Step	What to do
1	Write both equations and number them
2	Multiply so one variable has the same coefficient
3	Subtract if the signs are the same, add if opposite
4	Solve for one variable, then substitute to find the other
5	Check in both equations and state the answer in words

Elimination in five steps

Mistakes to expect

Learners often think	Put it right
It is enough to check in only one equation.	The solution must satisfy both equations at the same time, so check in both.
Always add the two equations.	Subtract when the coefficients have the same sign and add when they are opposites.
The answer is just x = 9 and y = 5.	State it in words, for example an exercise book costs K9 and a pen costs K5.

Differentiation

Learners who struggle

Give learners a model with the five steps filled in for one example, and let them copy the layout for the next. Keep the numbers small for the first two questions.

Learners who finish early

Ask learners who finish early to write their own shop problem with two unknowns, solve it, and swap with a partner to check.

Hearing or visual impairment

For learners with hearing impairment, write each step on the board and point to it as you explain. For learners with visual impairment, read each equation aloud slowly and let them speak the steps of working before writing in braille or large print.

No projector, no printing

Use prices from the school shop or market. All working is done on the board, and learners use their exercise books.

If time is short

Teach the five steps of elimination with the school shop example and one add-the-equations example. Leave the balance-idea equations as homework practice.

Evaluation

Question or task	A correct answer shows
At a school shop, 2 exercise books and 1 pen cost K23, and 1 exercise book and 2 pens cost K19. (a) Write two equations with x for the price of an exercise book and y for the price of a pen. (b) Solve them by elimination. (c) State the price of each item.	(a) 2x + y = 23 and x + 2y = 19. (b) Multiply the second by 2: 2x + 4y = 38. Subtract the first: 3y = 15, so y = 5, and then x = 9. (c) An exercise book costs K9 and a pen costs K5.
A learner buys 3 pencils and 1 ruler for K13 at a tuck shop in Chipata, and her friend buys 1 pencil and 1 ruler for K7. (a) Write two equations. (b) Solve by elimination. (c) Check in both equations.	(a) 3p + r = 13 and p + r = 7. (b) Subtract: 2p = 6, p = 3, then r = 4. (c) 3(3) + 4 = 13 and 3 + 4 = 7, both correct.
A trader sells tomatoes and onions: 2 kg of tomatoes and 3 kg of onions cost K42, and 3 kg of tomatoes and 2 kg of onions cost K43. (a) Write the two equations. (b) Say how to make the x coefficients equal. (c) Find the price per kilogram of each.	(a) 2x + 3y = 42 and 3x + 2y = 43. (b) Multiply the first by 3 and the second by 2. (c) Tomatoes K9 per kilogram and onions K8 per kilogram.

Homework

A learner at the market pays K14 for 3 kg of maize and 1 kg of groundnuts, and her mother pays K6 for 1 kg of maize and 1 kg of groundnuts. (a) Write two equations. (b) Solve them by elimination. (c) State the price per kilogram of each and check in both equations.

Extra practice with answers

Question	Answer
1. At a school stationery shop, 2 notebooks and 3 pens cost K19, and 1 notebook and 2 pens cost K11. (a) Write two equations. (b) Solve by elimination. (c) State the price of each.	(a) 2n + 3p = 19 and n + 2p = 11. (b) Multiply the second by 2: 2n + 4p = 22. Subtract the first: p = 3, then n = 5. (c) A notebook costs K5 and a pen costs K3.
2. A learner thinks of two numbers whose sum is 14 and whose difference is 6, and asks a friend to find them. (a) Write two equations. (b) Solve them. (c) Check in both.	(a) x + y = 14 and x - y = 6. (b) Add: 2x = 20, so x = 10 and y = 4. (c) 10 + 4 = 14 and 10 - 4 = 6, both correct.

3. A farmer buys 3 bags of fertiliser and 2 bags of seed for K24, and her neighbour buys 1 bag of fertiliser and 2 bags of seed for K12. (a) Write two equations. (b) Subtract to find the fertiliser price. (c) Find the seed price.	(a) 3x + 2y = 24 and x + 2y = 12. (b) 2x = 12, so x = 6. (c) 6 + 2y = 12, so y = 3.
4. At a bus stop, a learner buys 3 mangoes and 2 oranges for K17, and a friend buys 1 mango and 1 orange for K7 at the same stall. (a) Write two equations. (b) Solve by elimination. (c) State the price of each fruit.	(a) 3a + 2b = 17 and a + b = 7. (b) Multiply the second by 2 and subtract: a = 3, then b = 4. (c) A mango costs K3 and an orange costs K4.
5. A learner pays for two bus fares and a K5 snack and the total is K17. (a) Write the equation with x for one fare. (b) Solve it. (c) Check.	(a) 2x + 5 = 17. (b) 2x = 12, so x = 6. (c) 2(6) + 5 = 17, which is correct.
6. A pupil says that for x + y = 8 and x + 2y = 11 she should add the equations to eliminate x, and a friend disagrees. (a) Say who is right. (b) Solve the system. (c) Check the answer.	(a) The friend; the x coefficients have the same sign, so subtract. (b) y = 3 and x = 5. (c) 5 + 3 = 8 and 5 + 6 = 11, both correct.

Answers to the learners' check-yourself questions

1. What does a variable stand for in an equation?
   An unknown or changing quantity, shown by a symbol such as x or y.
2. Solve 4x - 3 = 21 and check your answer.
   4x = 24, so x = 6. Check: 4(6) - 3 = 21, which is correct.
3. Why must the same x and y be used in both equations of a system?
   Both equations describe the same two unknown quantities, and the solution must satisfy both at the same time.
4. Solve x + y = 10 and x - y = 4.
   Add the equations: 2x = 14, so x = 7. Then 7 + y = 10, so y = 3. Check: 7 - 3 = 4, which is correct.
5. You are a learner at a school tuck shop in Chipata. You buy 3 pencils and 1 ruler for K13. Your friend buys 1 pencil and 1 ruler for K7. (a) Write two equations, letting p be the price of a pencil and r the price of a ruler. (b) Solve them by elimination. (c) State the price of each item and check y
   (a) 3p + r = 13 and p + r = 7. (b) Subtract the second from the first: 2p = 6, so p = 3. Then 3 + r = 7, so r = 4. (c) A pencil costs K3 and a ruler costs K4. Check: 3(3) + 4 = 13 and 3 + 4 = 7, both correct.
6. You sell maize and beans at a market near Feira. One customer pays K30 for 2 kg of maize and 1 kg of beans. Another pays K35 for 1 kg of maize and 3 kg of beans. (a) Write two equations with m for maize and b for beans, both per kilogram. (b) Find the price per kilogram of each. (c) Explain what you
   (a) 2m + b = 30 and m + 3b = 35. (b) Multiply the second equation by 2: 2m + 6b = 70. Subtract the first: 5b = 40, so b = 8. Then 2m + 8 = 30, so m = 11. (c)

Maize costs K11 per kilogram and beans cost K8 per kilogram. Check: 2(11) + 8 = 30 and 11 + 3(8) = 35.

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: