Relations: types, arrow diagrams and ordered pairs

Lesson plan and teacher's notes · Form 2 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 2
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Relations: types, arrow diagrams and ordered pairs

Learners say what a relation between two sets is, name the four types of relation, show a relation as an arrow diagram and as ordered pairs, find set B from set A and a rule, and say when a relation is a function. The lesson uses learners and ID numbers, fathers and children, and number rules.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Explain what a relation between two sets is
2. Name the four types of relation: one-to-one, one-to-many, many-to-one and many-to-many
3. Show a relation using an arrow diagram and ordered pairs
4. Find the elements of set B for a given set A and relation
5. Say what a function is

Prior knowledge

- Sets written with curly brackets
- Adding and multiplying whole numbers
- Square roots of perfect squares

Teaching and learning materials

- chalk and board
- exercise books
- a ruler
- cards with the names of learners and their ID numbers

References

Ministry of Education (CDC) Form 2 Mathematics syllabus and module; 25Schools learners' note: Relations: types, arrow diagrams and ordered pairs.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Ask how a learner is linked to an ID number and how a teacher is linked to the learners they teach. Tell learners that mathematics calls these connections relations.	Give examples of things that are linked and copy the lesson title.
Development: what a relation is and its four types 12 min	Teach the meaning of a relation and the four types by looking at how many items are joined on each side. Use learners and ID numbers, is the father of and is the teacher of as examples.	Sort examples into one-to-one, one-to-many, many-to-one and many-to-many.
Development: arrow diagrams and ordered pairs 12 min	Work the example with set A = {1, 2, 3, 4} and the rule is multiplied by 2: draw the arrow diagram, find set B and write the ordered pairs with the element of A first.	Draw an arrow diagram and write ordered pairs for the rule add 3 on a set.
Development: functions 8 min	Teach that in a function each input has exactly one arrow. Work the square root example and then show why one-to-many and many-to-many relations are not functions.	Decide whether each relation is a function and give the reason.
Conclusion 3 min	Ask which two types of relation are functions. Set the homework.	Answer orally and copy the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

What is a relation

A relation describes how the elements of one set are connected or matched to the elements of another set. For example, the set of learners and the set of ID numbers are connected by the relation is the ID for. A relation can be shown as ordered pairs, as a table, as an arrow diagram (mapping) or as a graph. In this lesson we use ordered pairs and arrow diagrams.

The four types of relation

We name a relation by looking at how many items are joined on each side. In a one-to-one relation each item in the first set is linked to only one item in the second set, and each item in the second set is linked to only one in the first. Learners and their ID numbers are an example. In a one-to-many relation one item in the first set is linked to more than one item in the second set. Is the father of is an example, because one father can have several children. In a many-to-one relation several items in the first set are linked to the same item in the second set. In a many-to-many relation sev

Arrow diagrams and ordered pairs

An arrow diagram shows each set as a closed shape. An arrow goes from each element of set A to the element of set B that it is linked to. The same relation can be written as ordered pairs, with the element of A first and the element of B second. To find set B, apply the rule to every element of A.

Relations that are functions

A function is a special type of relation in which each input is linked to exactly one output. Look at the arrows from each element of set A. If every element has exactly one arrow leaving it, the relation is a function. So one-to-one and many-to-one relations are functions, because each input has only one arrow. One-to-many and many-to-many relations are not functions, because some input has more than one arrow.

Questions to ask during the lesson

Question	Expected answer
A headteacher says that every learner in the school is linked to an ID number and every teacher to the learners they teach. (a) Say what a relation between two sets is. (b) Name the four types of relation. (c) Name the type of relation that links learners and ID numbers.	(a) A connection that describes how the elements of one set are matched to the elements of another set. (b) One-to-one, one-to-many, many-to-one and many-to-many. (c) One-to-one.
A learner takes set A = {1, 2, 3} and the rule add 3 and wants to write the relation as ordered pairs. (a) Find set B. (b) Write the ordered pairs. (c) Say which element of each pair is written first.	(a) {4, 5, 6}. (b) (1, 4), (2, 5), (3, 6). (c) The element of set A.
A teacher says that one father can have several children and a learner can have many teachers. (a) Name the type of relation is the father of. (b) Name the type of relation is the teacher of. (c) Say whether a one-to-many relation is a function and explain.	(a) One-to-many. (b) Many-to-many. (c) No, because some input is linked to more than one output.
A learner looks at a relation in which each input has exactly one arrow leaving it. (a) Say what a function is. (b) Name the two types of relation that are functions. (c) Name the two types of relation that are not functions.	(a) A special relation in which each input is linked to exactly one output. (b) One-to-one and many-to-one. (c) One-to-many and many-to-many.

Worked examples for the board

Example 1: is multiplied by 2

Set A = {1, 2, 3, 4} and the relation is multiplied by 2. Find set B, write the ordered pairs and name the type of relation. Multiply each element of A by 2: 1 gives 2, 2 gives 4, 3 gives 6, 4 gives 8. Set B = {2, 4, 6, 8}. The ordered pairs are (1, 2), (2, 4), (3, 6), (4, 8). Each element of A has one arrow and each element of B receives one arrow, so the relation is one-to-one.

Example 2: is a square root of

Set A = {2, 3, 4, 5} and the relation is a square root of. Find set B, write the ordered pairs and say if the relation is a function. The square root of 4 is 2, of 9 is 3, of 16 is 4 and of 25 is 5, so set B = {4, 9, 16, 25}. The ordered pairs are (2, 4), (3, 9), (4, 16), (5, 25). Each element of A has exactly one arrow, so it is one-to-one. Because each input has exactly one output, it is a function.

Example 3: learners and clubs

Mulenga and Chileshe belong to the Science Club, while Thokozile and Nandi belong to the Drama Club. The ordered pairs are (Mulenga, Science), (Chileshe, Science), (Thokozile, Drama), (Nandi, Drama). Several learners are linked to the same club, so the relation is many-to-one. It is a function because each learner has exactly one club.

Board plan

Write the title and the key words. Left: the four types of relation with one example each. Centre: an arrow diagram with set A and set B and the ordered pairs below it. Right: the function test: does every input have exactly one arrow?

Type	Meaning	Function?
One-to-one	Each item in the first set links to one item in the second, and the reverse	Yes
One-to-many	One item in the first set links to more than one item in the second	No
Many-to-one	Several items in the first set link to the same item in the second	Yes
Many-to-many	Several items in the first set link to several items in the second	No

The four types of relation

Mistakes to expect

Learners often think	Put it right
In an ordered pair the elements can be written in any order.	An ordered pair is written (a, b), where a is from the first set and b is from the second. The element of set A is written first.
A one-to-many relation is a function.	A function is a relation in which each input is linked to exactly one output, so one-to-many and many-to-many relations are not functions because some input has more than one arrow.
A many-to-one relation is not a function because two items share an output.	Many-to-one relations are functions, because each input still has only one arrow. Several inputs may share the same output.
To find set B, apply the rule to only the first element of set A.	To find set B, apply the rule to every element of set A.

Differentiation

Learners who struggle

Give learners a blank arrow diagram with two ovals and the elements of set A already written. Do the first arrow together and ask learners to say the ordered pair for it before they draw the rest.

Learners who finish early

Ask learners who finish early to invent a relation between two sets from school life, draw its arrow diagram, name its type and say whether it is a function.

Hearing or visual impairment

For learners with hearing impairment, write each ordered pair on the board and point to the arrow it comes from. For learners with visual impairment, read each arrow aloud as from this element to that element, and let learners say the ordered pairs aloud.

No projector, no printing

Draw the ovals and arrows on the board with chalk. Learners copy them into exercise books and use pencil and ruler. Use names of learners in the class for the examples.

If time is short

Teach what a relation is, the four types and the arrow diagram with ordered pairs. Leave functions for the next lesson.

Evaluation

Question or task	A correct answer shows
At a school in Luangwa, Chanda has ID number 101, Mwila has 102 and Bwalya has 103, and no two learners share an ID number. (a) Write the ordered pairs for the relation is the ID for. (b) State the type of relation. (c) Say whether it is a function and why.	(a) (Chanda, 101), (Mwila, 102), (Bwalya, 103) or the pairs with the ID number first if the relation is read from ID to learner. (b) One-to-one. (c) Yes, because each input has exactly one output.
Mr Tembo is the father of Joyce, Peter and Ruth, and in this family no child has any other father in the set. (a) Write the ordered pairs for the relation is the father of. (b) State the type of relation. (c) Say whether it is a function and why.	(a) (Tembo, Joyce), (Tembo, Peter), (Tembo, Ruth). (b) One-to-many. (c) No, because the input Tembo is linked to more than one output.
A learner takes set A = {1, 2, 3, 4} and the relation add 5 and draws an arrow from each element to its output. (a) Find set B. (b) Write the ordered pairs. (c) State the type of relation and say whether it is a function.	(a) {6, 7, 8, 9}. (b) (1, 6), (2, 7), (3, 8), (4, 9). (c) One-to-one, and it is a function because each input has exactly one output.

Homework

At a school in Luangwa, Ben and Mary belong to the Red House and Joe belongs to the Blue House. A learner also draws the relation subtract 1 on the set A = {2, 3, 4}. (a) Write the ordered pairs for the houses and name the type of relation. (b) Find set B and

write the ordered pairs for subtract 1. (c) Say whether each relation is a function and why.

Extra practice with answers

Question	Answer
1. A learner at a school in Luangwa takes set A = {1, 2, 3, 4} and the relation is multiplied by 3 and draws an arrow from each element to its output. (a) Find set B. (b) Write the ordered pairs. (c) State the type of relation.	(a) {3, 6, 9, 12}. (b) (1, 3), (2, 6), (3, 9), (4, 12). (c) One-to-one.
2. At a shop, Chola and Mutale each buy sugar, and Bupe buys salt, with each customer buying only one item. (a) Write the ordered pairs for the relation buys. (b) State the type of relation. (c) Say whether it is a function and why.	(a) (Chola, sugar), (Mutale, sugar), (Bupe, salt). (b) Many-to-one. (c) Yes, because each customer has exactly one item.
3. At a school, Mrs Phiri teaches Ann and Joe, and Mr Zulu teaches Joe and Kay. A learner writes the relation from learners to teachers. (a) Write the ordered pairs with the learner first. (b) State the type of relation. (c) Say whether it is a function and why.	(a) (Ann, Phiri), (Joe, Phiri), (Joe, Zulu), (Kay, Zulu). (b) Many-to-many. (c) No, because Joe has two arrows leaving him.
4. A learner takes set A = {2, 4, 6} and the relation add 3 and draws an arrow diagram. (a) Find set B. (b) Write the ordered pairs. (c) Say whether the relation is a function and why.	(a) {5, 7, 9}. (b) (2, 5), (4, 7), (6, 9). (c) Yes, because each input has exactly one output.
5. A learner draws arrows from set A = {1, 2, 3} to set B = {5, 6}, with 1 and 2 both linked to 5 and 3 linked to 6. (a) Write the ordered pairs. (b) State the type of relation. (c) Say whether it is a function and why.	(a) (1, 5), (2, 5), (3, 6). (b) Many-to-one. (c) Yes, because each input has exactly one arrow.

Answers to the learners' check-yourself questions

1. What is a relation between two sets?
   A connection that describes how the elements of one set are matched to the elements of another set.
2. Name the four types of relation.
   One-to-one, one-to-many, many-to-one and many-to-many.
3. Write the relation 'add 3' from A = {1, 2, 3} as ordered pairs.
   (1, 4), (2, 5), (3, 6).
4. Is a one-to-many relation a function? Explain.
   No, because some input is linked to more than one output.
5. What type of relation is 'is the ID for' between learners and ID numbers?
   One-to-one.
6. You are a learner at a school in Luangwa. Four learners, Mulenga, Thokozile, Chileshe and Nandi, are each linked to the club they belong to: Mulenga and Chileshe belong to the Science Club, while Thokozile and Nandi belong to the Drama Club. (a) Draw in your book an arrow diagram for the relation 'b

(a) Arrows from Mulenga and Chileshe to Science Club, and from Thokozile and Nandi to Drama Club. (b) (Mulenga, Science), (Chileshe, Science), (Thokozile, Drama), (Nandi, Drama). (c) Many-to-one; it is a function because each learner has exactly one club.

7. A fish seller at a market in Feira sells bream and tilapia, and each buyer, Mr Banda, Mrs Zulu and Mr Phiri, buys from one or both. Mr Banda buys bream and tilapia, Mrs Zulu buys bream, and Mr Phiri buys tilapia. (a) Write the ordered pairs for 'buys'. (b) State the type of relation. (c) Explain why
   (a) (Banda, bream), (Banda, tilapia), (Zulu, bream), (Phiri, tilapia). (b) Many-to-many. (c) Mr Banda is linked to two items, so one input has more than one output.

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: