Expanding algebraic expressions

Lesson plan and teacher's notes · Form 2 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 2
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Expanding algebraic expressions

Learners say what monomials, binomials and quadratic expressions are, expand a monomial multiplied by a binomial, expand the product of two binomials using First, Outer, Inner, Last, and use expanded expressions to solve problems on cost and area. Examples come from Chanda buying pens and exercise books and a rectangular vegetable garden of a farmer in Chipata.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Say what monomials, binomials and quadratic expressions are
2. Expand a monomial multiplied by a binomial
3. Expand the product of two binomials using First, Outer, Inner, Last
4. Use expanded expressions to solve problems on cost and area

Prior knowledge

- Variables, constants, coefficients and terms from Form 1
- Multiplying whole numbers
- Combining like terms

Teaching and learning materials

- chalk and board
- exercise books
- a ruler for drawing a rectangle divided into parts

References

Ministry of Education (CDC) Form 2 Mathematics syllabus and module; 25Schools learners' note: Expanding algebraic expressions.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Ask how much 4 pens at k3 each and some exercise books at k8 each cost together. Tell learners that today they rewrite expressions in a different but equal form.	Suggest ways of writing the cost and copy the title.
Development: kinds of expressions 5 min	Define monomial, binomial, linear expression and quadratic expression with an example of each. Define expanding and simplification.	Copy the definitions and give one more example of each.
Development: monomial times binomial 12 min	Work Chanda's cost, 4(2x + 3), step by step: multiply 4 by each term inside the brackets, then add the results. Check by substituting a value for x.	Expand 3(x + 7) and two more brackets in their books.
Development: binomial times binomial 14 min	Teach FOIL with (x + 3)(x + 5). Then work the garden problem, with length (x + 5) and width (x + 2), writing First, Outer, Inner and Last each on a line before adding and simplifying.	Expand (x + 1)(x + 9) and a second pair of brackets using FOIL.
Conclusion 4 min	Ask what each letter in FOIL stands for. Set the homework.	Answer orally and copy the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

Kinds of expressions

A monomial is an expression with a single term, such as 4 or 3x. A binomial is an expression with two terms, such as 2x + 3. A linear expression is one where the variable has power 1, such as 8x + 12. A quadratic expression is one where the highest power of the variable is 2, such as $x^2 + 8x + 15$. Expanding means removing brackets by multiplying out. Simplification means writing an expression in a shorter equal form by combining like terms.

Monomial times binomial

To expand a single term (a monomial) times a two-term expression (a binomial), multiply each term inside the brackets by the single term. Chanda buys 4 pens at k3 each and some exercise books at k8 each. If she buys x books, the total cost is 8x + 12 kwacha. The same cost can be written as 4(2x + 3), and expanding that bracket takes us back to 8x + 12.

Binomial times binomial

To multiply two binomials, multiply every term in the first bracket by every term in the second. A helpful way to remember is FOIL: multiply the First terms, then the Outer terms, then the Inner terms, and last the Last terms. Then add and simplify like terms. A rectangular garden with length (x + m) metres and width (x + n) metres has area $x^2 + mx + nx + mn$.

Worked problem: a garden

A farmer in Chipata has a rectangular vegetable garden. The garden has length (x + 5) metres and width (x + 2) metres. The area is length times width, so (x + 5)(x + 2). First: x times x = x^2. Outer: x times 2 = 2x. Inner: 5 times x = 5x. Last: 5 times 2 = 10. Add: x^2 + 2x + 5x + 10. Combine like terms: $x^2 + 7x + 10$.

Questions to ask during the lesson

Question	Expected answer
A teacher writes 4, 3x, 2x + 3, 8x + 12 and $x^2 + 8x + 15$ on the board for a class in Luangwa. (a) Which of these is a monomial. (b) Which is a binomial. (c) Which is a quadratic expression.	(a) 4 or 3x. (b) 2x + 3 (also 8x + 12). (c) $x^2 + 8x + 15$.
Chanda buys 4 pens at k3 each and x exercise books at k8 each, and her cost can be written as 4(2x + 3). (a) Multiply 4 by 2x. (b) Multiply 4 by 3. (c) Write the expanded expression.	(a) 8x. (b) 12. (c) 8x + 12.
A learner expands (x + 1)(x + 9) in her book and wants to write each part correctly. (a) What does the letter F stand for in FOIL. (b) What do the letters O, I and L stand for. (c) Write the result after combining like terms.	(a) First. (b) Outer, Inner and Last. (c) $x^2 + 9x + x + 9 = x^2 + 10x + 9$.
A learner wants to check that the bracket form and the expanded form of Chanda's cost are equal by putting x = 5. (a) Find the value of 4(2x + 3) when x = 5. (b) Find the value of 8x + 12 when x = 5. (c) What does this show.	(a) 4(2 x 5 + 3) = 4 x 13 = 52. (b) 8 x 5 + 12 = 40 + 12 = 52. (c) The two forms are equal.

Worked examples for the board

Example 1: expand 4(2x + 3)

Expand 4(2x + 3). Multiply 4 by the first term: 4 x 2x = 8x. Multiply 4 by the second term: 4 x 3 = 12. Add the results: 8x + 12.

Example 2: expand (x + 3)(x + 5) with FOIL

First: x times x = x^2. Outer: x times 5 = 5x. Inner: 3 times x = 3x. Last: 3 times 5 = 15. Add: x^2 + 5x + 3x + 15. Combine like terms: $x^2 + 8x + 15$.

Example 3: the garden of a farmer in Chipata

The garden has length (x + 5) metres and width (x + 2) metres. Area = length times width = (x + 5)(x + 2). First: x times x = x^2. Outer: x times 2 = 2x. Inner: 5 times x = 5x. Last: 5 times 2 = 10. Add: x^2 + 2x + 5x + 10. Combine like terms: $x^2 + 7x + 10$.

Board plan

Write the title and key words. Left: the four kinds of expression with examples. Centre: Chanda's cost expanded step by step. Right: the garden worked with First, Outer, Inner and Last in four lines.

Step	Terms multiplied	Result
First	x times x	x^2
Outer	x times 5	5x
Inner	3 times x	3x
Last	3 times 5	15

Add and simplify $x^2 + 5x + 3x + 15$ $x^2 + 8x + 15$

FOIL for $(x + 3)(x + 5)$

Mistakes to expect

Learners often think	Put it right
In 4(2x + 3) only the first term inside the bracket is multiplied by 4.	Multiply each term inside the brackets by the single term, so 4 x 2x = 8x and 4 x 3 = 12, giving 8x + 12.
Expanding (x + 5)(x + 2) gives $x^2 + 10$.	Multiply every term in the first bracket by every term in the second: x^2, 2x, 5x and 10. Combine the like terms to get $x^2 + 7x + 10$.
A quadratic expression is any expression with two terms.	A quadratic expression is one where the highest power of the variable is 2, such as $x^2 + 8x + 15$. An expression with two terms is a binomial.
The expanded form and the bracket form give different answers.	They are equal. 4(2x + 3) and 8x + 12 give the same value for every x.

Differentiation

Learners who struggle

Give learners a grid with four boxes labelled First, Outer, Inner and Last so they can write each product in its own box before adding. Let them check each expansion by putting in x = 1.

Learners who finish early

Ask learners who finish early to write the area of a garden with length (x + 6) and width (x + 4) as an expanded expression, and to check it by choosing a value of x.

Hearing or visual impairment

For learners with visual impairment, read every term aloud and let them build the garden rectangle from string and cut pieces of card. For learners with hearing impairment, write each step of FOIL on the board and point to the terms as you multiply.

No projector, no printing

Draw the rectangle of the garden on the board or ground and split it into four parts to show x^2, 2x, 5x and 10. Use chalk only.

If time is short

Teach the kinds of expression and expand a monomial times a binomial with Chanda's cost. Leave binomial times binomial and the garden for the next lesson.

Evaluation

Question or task	A correct answer shows
Chanda buys 4 pens at k3 each and some exercise books at k8 each at a shop in Chipata. She buys x books, and the total cost is written as 4(2x + 3). (a) Multiply 4 by the first term. (b) Multiply 4 by the second term. (c) Write the expanded cost in kwacha.	(a) 4 x 2x = 8x. (b) 4 x 3 = 12. (c) 8x + 12.
A farmer in Chipata has a rectangular vegetable garden with length (x + 5) metres and width (x + 2) metres. (a) Write the area as a product. (b) Write the First, Outer, Inner and Last terms. (c) Write the area as an expanded expression.	(a) (x + 5)(x + 2). (b) x^2, 2x, 5x and 10. (c) x^2 + 7x + 10.
A learner at a school in Luangwa is asked about the expression 2x + 3 and the expression x^2 + 8x + 15. (a) Say what kind of expression 2x + 3 is. (b) Say what kind of expression x^2 + 8x + 15 is. (c) Explain how you know.	(a) A binomial, because it has two terms. (b) A quadratic expression. (c) The highest power of the variable is 2.

Homework

A trader at a market in Lusaka buys 3 sacks of maize meal and x bags of salt. Her cost in kwacha is written as 3(x + 7). A second trader has a stall that is (x + 1) metres long and (x + 9) metres wide. (a) Expand 3(x + 7). (b) Expand (x + 1)(x + 9). (c) Which of the two answers is a quadratic expression.

Extra practice with answers

Question	Answer
1. A shopkeeper in Chipata sells 5 pens at a price of (x + 4) kwacha each in one pack. (a) Write the total cost as a product. (b) Multiply 5 by each term in the bracket. (c) Write the expanded expression.	(a) 5(x + 4). (b) 5 x x = 5x and 5 x 4 = 20. (c) 5x + 20.
2. A learner at a school in Luangwa expands 2(3x + 1) for homework. (a) Multiply 2 by 3x. (b) Multiply 2 by 1. (c) Write the final expression.	(a) 6x. (b) 2. (c) 6x + 2.
3. A farmer near Feira has a rectangular field with length (x + 6) metres and width (x + 2) metres. (a) Write the area as a product. (b) Write the First, Outer, Inner and Last terms. (c) Write the area as an expanded expression.	(a) (x + 6)(x + 2). (b) x^2, 2x, 6x and 12. (c) x^2 + 8x + 12.

4. A farmer in Chipata finds that the garden with length (x + 5) metres and width (x + 2) metres has area $x^2 + 7x + 10$, and she tries x = 3 to check it. (a) Find the length and width when x = 3. (b) Find the area from the product. (c) Find $x^2 + 7x + 10$ when x = 3 and compare.

(a) Length 8 m and width 5 m. (b) 8 x 5 = 40 square metres. (c) 9 + 21 + 10 = 40, so the two are equal.

Answers to the learners' check-yourself questions

1. What is a binomial? Give one example.
   An expression with two terms, for example 2x + 3.
2. Expand 3(x + 7).
   3x + 21.
3. Expand (x + 1)(x + 9).
   x2 + 9x + x + 9 = x2 + 10x + 9.
4. What does the letter F stand for in FOIL?
   Multiply the First terms.
5. Expand (x + 2)(x + 6).
   x2 + 6x + 2x + 12 = x2 + 8x + 12.
6. A group of students at a school near Feira hires 2x taxis to travel to Lusaka. Each taxi charges (y + 50) kwacha. (a) Write the total cost as a product. (b) Expand it. (c) Find the cost when x = 3 and y = 100.
   (a) 2x(y + 50). (b) 2xy + 100x. (c) 2xy + 100x = 2(3)(100) + 100(3) = 600 + 300 = K900.
7. A farmer's rectangular garden has length (x + 5) metres and width (x + 4) metres. (a) Write the area as an expanded expression. (b) Find the area when x = 3. (c) Check your answer by multiplying the length and width directly.
   (a) x2 + 4x + 5x + 20 = x2 + 9x + 20. (b) 9 + 27 + 20 = 56 square metres. (c) (3 + 5)(3 + 4) = 8 x 7 = 56 square metres.

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: