Pythagoras theorem

Lesson plan and teacher's notes · Form 1 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 1
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Pythagoras theorem

Learners identify the hypotenuse of a right-angled triangle, state the Pythagoras theorem, find a missing side, and use the theorem to solve real-life problems. Examples come from a rectangular field with a shortcut across it, a ladder leaning on a wall and fencing a piece of land.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Identify the hypotenuse of a right-angled triangle
2. State the Pythagoras theorem
3. Find a missing side of a right-angled triangle
4. Use the theorem to solve real-life problems

Prior knowledge

- A right angle is 90 degrees
- Squares of whole numbers, such as 6 x 6 = 36
- Square roots of simple perfect squares

Teaching and learning materials

- squared paper
- rulers
- chalk and board
- exercise books
- a broomstick or a plank to lean on a wall

References

Ministry of Education (CDC) Form 1 Mathematics syllabus and module; 25Schools learners' note: Pythagoras theorem.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Ask whether it is shorter to walk across a rectangular field from one corner to the opposite corner, or to go round the edges. Tell learners that the Pythagoras theorem gives the exact length of the shortcut.	Give opinions and copy the title and key words.
Development: the sides of a right-angled triangle 8 min	Lean a broomstick or draw a ladder against a wall. Show that the wall, the floor and the ladder form a right-angled triangle, and that the ladder is the longest side, the hypotenuse.	Draw a right-angled triangle and mark the right angle and the hypotenuse.
Development: the rule 10 min	On squared paper, draw the triangle with sides 6 cm, 8 cm and 10 cm and a square on each side. Show that 36 + 64 = 100. State $c^2 = a^2 + b^2$ and check it with 5, 12 and 13.	Draw the triangle and squares, and copy the rule.
Development: solving problems 14 min	Work the hypotenuse example with 6 cm and 8 cm. Then work the ladder example, in which you subtract: $b^2 = c^2 - a^2$. Stress drawing a sketch first and marking the right angle.	Solve a hypotenuse problem and a missing-side problem in their books.
Conclusion 3 min	Ask when you add the squares and when you subtract them. Set the homework.	Answer orally and copy the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

The sides of a right-angled triangle

The theorem is named after the ancient Greek mathematician Pythagoras. It describes a special relationship between the sides of a right-angled triangle. A right-angled triangle has one angle of 90 degrees. You can see the shape when a ladder or a broomstick leans against a wall. The wall, the floor and the ladder form a right-angled triangle, and the ladder is the longest side, the hypotenuse. The hypotenuse is opposite the right angle.

The rule $c^2 = a^2 + b^2$

Draw a right-angled triangle with sides 6 cm, 8 cm and 10 cm on squared paper, and draw a square on each side. The squares have areas 36, 64 and 100 square centimetres. The two smaller squares add up to 36 + 64 = 100, which is the area of the biggest square. This is the Pythagoras theorem. If a and b are the two shorter sides and c is the hypotenuse, then $c^2 = a^2 + b^2$. In words, the square on the hypotenuse equals the sum of the squares on the other two sides. You can check it with other triangles, for example 5, 12 and 13, where 25 + 144 = 169.

Finding the hypotenuse

To find the hypotenuse, add the squares of the two shorter sides and take the square root. A right-angled triangle has shorter sides 6 cm and 8 cm. $c^2 = 6 \times 6 + 8 \times 8 = 36 + 64 = 100$, so c = square root of 100 = 10 cm.

Finding a shorter side

To find a shorter side, subtract: $b^2 = c^2 - a^2$, then take the square root. A ladder 13 m long leans against a wall with its foot 5 m from the wall. The ladder is the hypotenuse, so c = 13 m and one side a = 5 m. $b^2 = 169 - 25 = 144$, so b = square root of 144 = 12 m. Always draw a sketch and mark the right angle first so you know which side is the hypotenuse.

Questions to ask during the lesson

Question	Expected answer
A learner at a school in Luangwa sees a ladder leaning against a wall, with the wall, the floor and the ladder forming a triangle. (a) What kind of triangle is this. (b) Which side is the hypotenuse. (c) Where is the hypotenuse in relation to the right angle.	(a) A right-angled triangle. (b) The ladder, the longest side. (c) Opposite the right angle.
A teacher draws a triangle with sides 6 cm, 8 cm and 10 cm on squared paper with a square on each side. (a) Give the area of each of the three squares. (b) Add the two smaller areas. (c) What does this show about the square on the hypotenuse.	(a) 36, 64 and 100 square centimetres. (b) 36 + 64 = 100. (c) It equals the sum of the squares on the other two sides.
A farmer wants to find the shortcut across her field using the sides 6 m and 8 m. (a) Write the rule using a, b and c. (b) Find c^2. (c) Find c.	(a) $c^2 = a^2 + b^2$. (b) 36 + 64 = 100. (c) 10 m.
A builder knows a ladder is 13 m long and its foot is 5 m from the wall, and wants the height reached. (a) Which is the hypotenuse. (b) Do you add or subtract the squares. (c) Find the height.	(a) The ladder, 13 m. (b) Subtract: $b^2 = c^2 - a^2$. (c) 169 - 25 = 144, so b = 12 m.

Worked examples for the board

Example 1: find the hypotenuse

A right-angled triangle has shorter sides 6 cm and 8 cm. Find the hypotenuse. $c^2 = a^2 + b^2$. $c^2 = 6 \times 6 + 8 \times 8 = 36 + 64$. $c^2 = 100$. c = square root of 100 = 10 cm.

Example 2: the ladder against a wall

A ladder 13 m long leans against a wall with its foot 5 m from the wall. How high up the wall does it reach? The ladder is the hypotenuse, so c = 13 m and one side a = 5 m. $b^2 = c^2 - a^2 = 169 - 25$. $b^2 = 144$. b = square root of 144 = 12 m.

Example 3: check 5, 12 and 13

Take a triangle with sides 5, 12 and 13. The square of 5 is 25 and the square of 12 is 144. The sum is 25 + 144 = 169. The square of 13 is 169, so the rule works for this triangle too.

Board plan

Write the title and key words. Left: a right-angled triangle with the hypotenuse marked. Centre: the squares 36, 64 and 100 on the 6, 8, 10 triangle and the rule $c^2 = a^2 + b^2$. Right: the two worked examples, one adding and one subtracting.

Triangle sides	$a^2 + b^2$	c^2
6 cm, 8 cm and 10 cm	36 + 64 = 100	10 x 10 = 100
5, 12 and 13	25 + 144 = 169	13 x 13 = 169

Checking the rule on triangles

Mistakes to expect

Learners often think	Put it right
Any side can be called the hypotenuse.	The hypotenuse is the longest side of a right-angled triangle, opposite the right angle.
To find a shorter side you add the squares.	To find the hypotenuse you add the squares of the two shorter sides. To find a shorter side you subtract: $b^2 = c^2 - a^2$.
The answer is 100 once $c^2 = 100$.	$c^2 = 100$ is the square of the side, so take the square root. c = 10 cm.
There is no need to draw a sketch.	Always draw a sketch and mark the right angle first so you know which side is the hypotenuse.

Differentiation

Learners who struggle

Give learners a card with the two steps: add the squares to find the hypotenuse, subtract to find a shorter side. Let them label a, b and c on a printed triangle before they calculate.

Learners who finish early

Ask learners who finish early to check whether 7, 24 and 25 satisfy $c^2 = a^2 + b^2$, and to make up a ladder problem with a hypotenuse of 13 m.

Hearing or visual impairment

For learners with visual impairment, make the triangle from string or sticks so they can feel the right angle and the longest side. For learners with hearing impairment, write each step of the working on the board in order.

No projector, no printing

Draw the triangle and squares on the ground or board if squared paper is not available. Use a broomstick leaning on a wall for the ladder.

If time is short

Teach the hypotenuse and the rule with the 6, 8, 10 triangle, then work the hypotenuse example. Leave finding a shorter side and the ladder problem for the next lesson.

Evaluation

Question or task	A correct answer shows
A farmer near Feira has a rectangular field and wants to know the shortcut across it. Two shorter sides of a right-angled triangle on her sketch are 6 m and 8 m. (a) Name the longest side of the triangle. (b) Write the rule for the hypotenuse. (c) Find the length of the shortcut.	(a) The hypotenuse. (b) $c^2 = a^2 + b^2$. (c) $c^2 = 36 + 64 = 100$, so c = 10 m.
A ladder 13 m long leans against a wall at a school in Luangwa with its foot 5 m from the wall. (a) Say which side of the triangle the ladder is. (b) Write the calculation for the height reached. (c) Find the height.	(a) The hypotenuse. (b) $b^2 = 13^2 - 5^2 = 169 - 25$. (c) $b^2 = 144$, so b = 12 m.
A learner in Lusaka draws a triangle with sides 5, 12 and 13 to check the theorem. (a) Find the sum of the squares of the two shorter sides. (b) Find the square of the longest side. (c) Say what this shows.	(a) 25 + 144 = 169. (b) 13 x 13 = 169. (c) The square on the hypotenuse equals the sum of the squares on the other two sides.

Homework

A builder at a school in Chipata fences a piece of land with a right-angled corner. The two shorter sides of the corner are 3 m and 4 m, and a ladder 10 m long leans against a wall with its foot 6 m from the wall. (a) Find the length of the diagonal across the corner. (b) Find how high the ladder reaches. (c) Say in which of the two you add the squares and in which you subtract.

Extra practice with answers

Question	Answer
1. A farmer near Feira measures a corner of her field where the two shorter sides are 6 m and 8 m. (a) Write the rule. (b) Find c^2. (c) Find the length of the hypotenuse.	(a) $c^2 = a^2 + b^2$. (b) 36 + 64 = 100. (c) 10 m.
2. A ladder 13 m long leans against a wall at a school with its foot 5 m from the wall. (a) Which side is the ladder. (b) Find b^2. (c) How high does the ladder reach.	(a) The hypotenuse. (b) 169 - 25 = 144. (c) 12 m.
3. A builder in Lusaka finds the shorter sides of a right-angled corner are 3 m and 4 m. (a) Find 3 x 3 and 4 x 4. (b) Add them to find c^2. (c) Find c.	(a) 9 and 16. (b) 9 + 16 = 25. (c) 5 m.

4. A ladder 10 m long leans against a wall with its foot 6 m from the wall. (a) Which side is 10 m. (b) Find $b^2 = c^2 - a^2$. (c) Find the height reached.

(a) The hypotenuse. (b) 100 - 36 = 64. (c) 8 m.

Answers to the learners' check-yourself questions

1. What is the hypotenuse?
   The longest side of a right-angled triangle, opposite the right angle.
2. State the Pythagoras theorem.
   c2 = a2 + b2, where c is the hypotenuse.
3. A right-angled triangle has shorter sides 3 cm and 4 cm. Find the hypotenuse.
   c2 = 9 + 16 = 25, so c = 5 cm.
4. A right-angled triangle has hypotenuse 17 cm and one side 8 cm. Find the other side.
   b2 = 289 - 64 = 225, so b = 15 cm.
5. Is a triangle with sides 5 cm, 12 cm and 13 cm right-angled? Show your working.
   5 x 5 + 12 x 12 = 25 + 144 = 169 = 13 x 13, so yes, it is right-angled.
6. In a village near Feira, Mr Mwansa wants to fence a right-angled triangular maize field with shorter sides 30 m and 40 m. (a) Find the length of the third side. (b) Find the total length of fence needed. (c) Find the cost of the fence at K12 per metre.
   (a) c2 = 900 + 1600 = 2500, so c = 50 m. (b) 30 + 40 + 50 = 120 m. (c) 120 x 12 = K1440.
7. A rectangular field is 60 m long and 80 m wide. A learner walks along the two edges from one corner to the opposite corner, and her friend walks straight across along the diagonal. (a) Find the length of the diagonal. (b) Find how far the learner walks along the edges. (c) Say how much shorter the d
   (a) c2 = 3600 + 6400 = 10000, so c = 100 m. (b) 60 + 80 = 140 m. (c) 140 - 100 = 40 m shorter.

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: