Matrices: adding, subtracting and multiplying in real life

Lesson plan and teacher's notes · Form 1 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 1
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Matrices: adding, subtracting and multiplying in real life

Learners write information from real situations as matrices, add and subtract matrices of the same order, multiply a matrix by a scalar, multiply a matrix by a matrix up to 2 by 2, and show that matrix multiplication is not commutative. All work is set in budgets, shops and concert costs.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Write information from a real situation in matrix form
2. Add and subtract matrices of the same order
3. Multiply a matrix by a constant (scalar)
4. Multiply a matrix by a matrix, up to 2 by 2, and say when it is possible
5. Explain that matrix multiplication is not commutative

Prior knowledge

- Reading a table of numbers by rows and columns
- Addition, subtraction and multiplication of whole numbers
- Negative numbers

Teaching and learning materials

- chalk and board
- exercise books
- a table of weekly budgets drawn on the board
- pencils

References

Ministry of Education (CDC) Form 1 Mathematics syllabus and module; 25Schools learners' note: Matrices: adding, subtracting and multiplying in real life.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Draw the weekly budget table for three districts and three departments. Ask where rows and columns are in the table. Tell learners that the nine numbers kept in their positions form a matrix.	Read the table and say what each row and each column stands for.
Development: matrix form, adding and subtracting 12 min	Teach order (rows by columns) and element. Work the Week 1 and Week 2 budget example: add elements in the same positions to get the total and subtract to get the change. State that the matrices must have the same order.	Write the order of each matrix and add and subtract the two budget matrices.
Development: scalar multiplication 8 min	Work the five concerts example: multiply every element of the cost column by 5.	Multiply each element of a column by a scalar and say the order of the answer.
Development: multiplying matrices 12 min	State the rule: columns of the first equal rows of the second. Work the Kitwe and Lusaka stock and price example row by column, then multiply two matrices in both orders to show the answers differ.	Check whether two matrices can be multiplied and multiply a 2 by 2 pair in both orders.
Conclusion 3 min	Ask when two matrices can be added and when they can be multiplied. Set the homework.	Answer orally and copy the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

Writing information as a matrix

Rows go across and columns go down. A matrix with 3 rows and 3 columns has order 3 by 3. To form a matrix from a situation, keep the same position for the same kind of information. In the budget table each row is a district and each column is a department, and the matrix is just the nine numbers kept in the same positions.

Adding and subtracting matrices

To add or subtract two matrices, they must be of the same order. Then add or subtract the elements that are in the same positions. For the two weekly budgets, adding gives the total over the two weeks and subtracting gives the change from Week 1 to Week 2.

Multiplying a matrix by a scalar

To multiply a matrix by a number, multiply every element by that number. If one concert costs K1,200 for the venue, K800 for equipment, K500 for staff and security and K300 for transport, written as a column, then for 5 concerts we multiply each entry by 5.

Multiplying a matrix by a matrix

Two matrices can be multiplied only when the number of columns in the first equals the number of rows in the second. To get an element of the answer, multiply the elements of a row of the first matrix by the matching elements of a column of the second, and add the products. Matrix multiplication is not commutative: if A and B can be multiplied both ways, A times B and B times A usually give different answers.

Questions to ask during the lesson

Question	Expected answer
A district office keeps a table with 3 rows for districts and 2 columns for departments. (a) State the order of the matrix. (b) Say what an element is. (c) Say which direction rows go and which direction columns go.	(a) 3 by 2. (b) One number in a matrix. (c) Rows go across and columns go down.
A shop owner has two weekly stock tables P = [3 5; 2 4] and Q = [1 0; 6 2]. (a) Find P + Q. (b) Find P − Q. (c) Say when two matrices can be added.	(a) [4 5; 8 6]. (b) [2 5; −4 2]. (c) When they are of the same order.
A band plans three times its costs for a tour, and the costs for one tour are in the matrix [2 4; 1 5]. (a) Find 3 times the matrix. (b) Say what you did to each element. (c) Say what a scalar is.	(a) [6 12; 3 15]. (b) Multiplied every element by 3. (c) An ordinary number that multiplies every element of a matrix.
A learner has [1 2; 3 4] and a column [5; 6] and wants to multiply them. (a) Say whether it can be done. (b) Give the rule for multiplying two matrices. (c) Say whether A times B is always equal to B times A.	(a) Yes, the first has 2 columns and the second has 2 rows, and the answer is [17; 39]. (b) The number of columns in the first equals the number of rows in the second. (c) No, matrix multiplication is not commutative.

Worked examples for the board

Example 1: two weeks of budgets

Week 1 budget A is [120 90 75; 100 85 60; 110 95 70] and Week 2 budget B is [130 85 80; 95 88 65; 115 92 75], in thousands of kwacha. Find C, the total over the two weeks, and the change from Week 1 to Week 2. Both are 3 by 3, so we can add them. Row 1: 120+130 = 250, 90+85 = 175, 75+80 = 155. Row 2: 100+95 = 195, 85+88 = 173, 60+65 = 125. Row 3: 110+115 = 225, 95+92 = 187, 70+75 = 145. So C = [250 175 155; 195 173 125; 225 187 145]. The change B − A has rows 10, −5, 5; −5, 3, 5; and 5, −3, 5. Funding fell for Health in District 1, Education in District 2 and Health in District 3.

Example 2: five concerts

The costs for one concert are [1200; 800; 500; 300]. Multiply every element by 5: 1200 × 5 = 6000, 800 × 5 = 4000, 500 × 5 = 2500 and 300 × 5 = 1500. The costs are [6000; 4000; 2500; 1500].

Example 3: stock and prices

A shop has stock X = [30 50; 40 35], with rows Kitwe and Lusaka and columns calculators and mathematical sets. The prices are Y = [70 65 60; 50 45 40], with rows calculators and sets and columns standard, discounted and wholesale price in kwacha. X has 2 columns and Y has 2 rows, so X × Y can be found. Kitwe: 30×70 + 50×50 = 4600, 30×65 + 50×45 = 4200, 30×60 + 50×40 = 3800. Lusaka: 40×70 + 35×50 = 4550, 40×65 + 35×45 = 4175, 40×60 + 35×40 = 3800. So X × Y = [4600 4200 3800; 4550 4175 3800].

Board plan

Write the title and the key words. Left: the Week 1 budget table and the matrix with its order. Centre: the add and subtract working row by row. Right: the stock and price multiplication with the row-by-column products.

Operation	Rule	Example from the lesson
Add or subtract	Same order, then add or subtract elements in the same positions	A + B gives the total budget over two weeks
Multiply by a scalar	Multiply every element by the number	5 concerts: every cost multiplied by 5
Multiply two matrices	Columns of the first equal rows of the second	Stock X times prices Y gives the value in each branch
Order of multiplication	A times B and B times A usually differ	Not commutative

Rules for matrix operations

Mistakes to expect

Learners often think	Put it right
Any two matrices can be added.	Two matrices can be added or subtracted only when they are of the same order. Then the elements in the same positions are added or subtracted.
To multiply by a scalar, multiply only the first element.	Multiply every element by the number. For 5 concerts every cost in the column is multiplied by 5.
Two matrices can be multiplied whenever they are the same size.	Two matrices can be multiplied only when the number of columns in the first equals the number of rows in the second.
A times B always gives the same answer as B times A.	Matrix multiplication is not commutative. If A and B can be multiplied both ways, A times B and B times A usually give different answers.

Differentiation

Learners who struggle

Give learners the table of rules and a small 2 by 2 grid to fill in. For multiplication, ask them to point to the row of the first matrix and the column of the second with two fingers before they calculate each element.

Learners who finish early

Ask learners who finish early to write their own stock and price example with two branches and two items, and to find the value of stock in each branch at two prices.

Hearing or visual impairment

For learners with visual impairment, read each matrix aloud row by row and let learners say the working aloud as well. For learners with hearing impairment, write each step on the board and point to the row and column being multiplied.

No projector, no printing

Draw every table and matrix on the board with chalk. Learners copy them into exercise books and use the numbers from the note, so no printed material or calculator is needed.

If time is short

Teach matrix form, adding and subtracting, and scalar multiplication. Leave multiplying a matrix by a matrix and non-commutativity for the next lesson.

Evaluation

Question or task	A correct answer shows
Two bakery stalls in Kitwe sell loaves and buns. On Monday they sold M = [40 25; 30 35] and on Tuesday T = [35 30; 45 20], with rows for Stall P and Stall Q and columns for loaves and buns. (a) Find M + T. (b) Find T − M. (c) Say which stall and which item sold fewer on Tuesday than on Monday.	(a) [75 55; 75 55]. (b) [−5 5; 15 −15]. (c) Stall P loaves (down by 5) and Stall Q buns (down by 15).
A stall owner in Lusaka has stock S = [6 3; 2 5] and orders four times that stock again from a supplier. (a) Write the matrix for the order. (b) Find the stock after adding the order to S. (c) Say how this answer is related to S as a scalar.	(a) 4S = [24 12; 8 20]. (b) S + 4S = [30 15; 10 25]. (c) It is S multiplied by the scalar 5.
A learner multiplies two matrices from a stock table, A = [2 1; 3 4] and B = [1 0; 2 5], first as A × B and then as B × A. (a) Find A × B. (b) Find B × A. (c) Say what the two answers show.	(a) [4 5; 11 20]. (b) [2 1; 19 22]. (c) They are different, so matrix multiplication is not commutative.

Homework

A wholesaler has stock X = [20 10; 15 25], with rows for the Chipata and Mansa branches and columns for bags of maize meal and bags of sugar. The prices are Y = [8 7; 5 4], with rows for meal and sugar and columns for standard and discounted prices in kwacha. (a) Say why X × Y can be found. (b) Find X × Y. (c) Say what the first row of the answer tells the wholesaler.

Extra practice with answers

Question	Answer
1. A school has two tuck-shop stalls. On Monday they sold [25 14; 18 30] and on Tuesday [20 16; 22 28], with rows for Stall 1 and Stall 2 and columns for buns and sodas. (a) Find the total for the two days. (b) Find the change from Monday to Tuesday. (c) Say which stall and item fell.	(a) [45 30; 40 58]. (b) [−5 2; 4 −2]. (c) Stall 1 buns and Stall 2 sodas.
2. A band in Lusaka pays [900; 600; 400; 200] for the venue, equipment, staff and transport for one concert and plans to hold 3 concerts. (a) Write the matrix of costs for 3 concerts. (b) Say what you did to each element. (c) Find the total cost of the 3 concerts.	(a) [2700; 1800; 1200; 600]. (b) Multiplied every element by 3. (c) K6,300.
3. A learner reads the matrix [4 1 7; 2 0 9] from a stock table and wants to add it to a matrix with 3 rows and 2 columns. (a) State the order of the first matrix. (b) Name the element in row 2, column 3. (c) Say whether the two matrices can be added and why.	(a) 2 by 3. (b) 9. (c) No, because they are not of the same order.
4. A shop has stock X = [10 5; 8 6], with rows Kitwe and Lusaka and columns pens and books, and prices Y = [3 2; 4 5], with rows pens and books and columns standard and discounted. (a) Say why X × Y can be found. (b) Find X × Y. (c) Say what the Lusaka row tells the owner.	(a) X has 2 columns and Y has 2 rows. (b) [50 45; 48 46]. (c) The value of Lusaka stock at the standard and discounted prices, K48 and K46.
5. A learner multiplies A = [1 2; 0 3] and B = [2 0; 1 1] in both orders and compares the answers. (a) Find A × B. (b) Find B × A. (c) Say what the comparison shows.	(a) [4 2; 3 3]. (b) [2 4; 1 5]. (c) The answers are different, so matrix multiplication is not commutative.

Answers to the learners' check-yourself questions

1. State the order of a matrix with 2 rows and 3 columns.
   2 by 3.
2. When can two matrices be added?
   When they are of the same order.
3. Find 4 × [2 5; 1 3].
   [8 20; 4 12].
4. Find [1 2; 3 4] + [5 0; 2 1].
   [6 2; 5 5].
5. Given A = [1 2; 3 4] and B = [0 1; 1 0], find A × B and B × A. What do the answers show?
   A × B = [2 1; 4 3] and B × A = [3 4; 1 2]. They are different, so matrix multiplication is not commutative.
6. A shop (kantemba) receives deliveries of milk, bread and vegetables in crates from two suppliers. Week 1 is W = [10 8 6; 12 9 5] and Week 2 is K = [11 7 8; 10 12 6], with rows for Supplier A and B. (a) Find W + K. (b) Find K − W. (c) Which supplier and item had a reduction from Week 1 to Week 2?

(a) W + K = [21 15 14; 22 21 11]. (b) K − W = [1 −1 2; −2 3 1]. (c) Supplier A bread (down by 1) and Supplier B milk (down by 2).

7. A school tuck shop records its stock as Q = [12 8; 20 10], with rows Monday and Tuesday and columns buns and sodas. A new delivery doubles every number. (a) Write the matrix for the delivery. (b) Find the stock after adding the delivery to Q. (c) Explain how you got (b) in terms of a scalar.
   (a) 2Q = [24 16; 40 20]. (b) Q + 2Q = [36 24; 60 30]. (c) Adding Q and 2Q is the same as multiplying Q by the scalar 3.

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: