Approximations and estimation

Lesson plan and teacher's notes · Form 1 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 1
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Approximations and estimation

Learners estimate answers by rounding, round to significant figures and decimal places, write large and small numbers in standard form, and find absolute, relative and percentage error and compare it with a tolerance. The lesson uses a class trip, a harvest of mangoes and a carpenter's plank.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Estimate an answer in a real situation by rounding each number first
2. Round a number to a given number of significant figures or decimal places
3. Write a large or small number in standard form
4. Find absolute, relative and percentage error and decide whether a measurement is within a tolerance

Prior knowledge

- Place value and rounding to the nearest ten or hundred
- Multiplying and dividing whole numbers
- Powers of 10

Teaching and learning materials

- chalk and board
- a measuring tape or ruler
- a piece of wood or a pencil to measure
- exercise books

References

Ministry of Education (CDC) Form 1 Mathematics syllabus and module; 25Schools learners' note: Approximations and estimation.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Ask how a teacher could quickly guess how many tests are marked in a year when 31 pupils write one test a week. Write estimate and approximate on the board.	Suggest quick ways to get a rough answer and say why exactness may not matter.
Development: estimating 10 min	Work the 31 pupils and 39 weeks example: 30 × 40 = 1,200. Work the class trip: round each cost and find about K330 each.	Round and estimate two more examples, then compare with a calculator.
Development: rounding and standard form 12 min	Teach how to count significant figures from the first non-zero digit and how to round. Write 384,000,000 as 3.84 × 10^8 and compare 2.3 × 10^-8 with 5.1 × 10^-7.	Round three numbers and write three numbers in standard form.
Development: error and tolerance 10 min	Use the carpenter: actual 100 cm, measured 98 cm. Find absolute error, relative error, percentage error and compare with a tolerance of ±1 cm.	Measure a real length, compare with a known value and find the errors.
Conclusion 3 min	Ask why a tolerance is useful. Set the homework.	Answer and copy the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

Estimating

To estimate, round each number to a simple value and calculate with the rounded values. The answer is not exact but shows the size of the exact answer. For 31 tests a week over 39 weeks, 30 × 40 = 1,200, so about 1,200 tests are marked.

Rounding

Round to the nearest unit, fraction, decimal place or significant figure. Look at the digit after the last one you keep. If it is 5 or more, increase the last kept digit by 1. If it is less than 5, leave it.

Significant figures

Count from the first non-zero digit. In 0.005709 the first significant figure is 5, so it has 4 significant figures. Fill any places up to the decimal point with zeros when rounding a large number.

Standard form

Write a number as a × 10 to a power, where a is between 1 and 10. For example 384,000,000 = 3.84 × 10^8, and rounded to 2 significant figures it is 3.8 × 10^8. A number with a smaller negative power, such as 10^-8, is smaller than one with 10^-7.

Error and tolerance

Absolute error is the difference between measured and actual values, taken as positive. Relative error is the absolute error divided by the actual value, and multiplying by 100 gives the percentage error. A measurement is acceptable when its absolute error is within the tolerance.

Questions to ask during the lesson

Question	Expected answer
A class of 15 learners plans a trip where the bus costs K485, lunch is K95 per learner and entrance is K175 per learner. (a) Round each cost. (b) Estimate the total for the class. (c) Estimate how much each learner pays.	(a) K485 to K500, K95 to K100 and K175 to K200. (b) 15 × 300 = K4,500, and with the bus K5,000. (c) About 5,000 ÷ 15, which is about K330.
A learner reads the number 0.005709 on a scale and her teacher asks how many significant figures it has. (a) Say where counting starts. (b) State the number of significant figures. (c) Round it to 2 significant figures.	(a) At the first non-zero digit. (b) 4 significant figures. (c) 0.0057.
A shopkeeper counts 9 biscuits in each of 41 bags and wants a quick estimate of the total. (a) Round the numbers. (b) Find the estimate. (c) Find the exact total and the absolute error.	(a) 9 to 10 and 41 to 40. (b) 10 × 40 = 400. (c) 9 × 41 = 369, so the absolute error is |400 − 369| = 31.
A learner measures a table at 98 cm when the actual length is 100 cm. (a) Find the absolute error. (b) Find the relative error. (c) Give the percentage error.	(a) |98 − 100| = 2 cm. (b) 2 ÷ 100 = 0.02. (c) 2%.

Worked examples for the board

Example 1: class trip

15 learners go on a trip. The bus costs K485, lunch is K95 per learner and entrance is K175 per learner. Round to K500, K100 and K200. Cost per learner for lunch and entrance = 100 + 200 = K300. For 15 learners 15 × 300 = K4,500, and with the bus 4,500 + 500 = K5,000. Each pays about 5,000 ÷ 15, about K330.

Example 2: standard form

384,000,000 metres = 3.84 × 10^8 metres. To 2 significant figures it is 3.8 × 10^8. A length of 4,560,000 is 4.56 × 10^6, and 0.00072 is 7.2 × 10^-4.

Example 3: error and tolerance

A plank should be 100 cm and measures 98 cm. Absolute error = |98 − 100| = 2 cm. Relative error = 2 ÷ 100 = 0.02. Percentage error = 0.02 × 100% = 2%. With tolerance ±1 cm, 2 cm is outside it, so it is not acceptable.

Board plan

Write the title. Left: the rules for estimating by rounding and for counting significant figures. Centre: standard form examples. Right: error steps in order: absolute error, relative error, percentage error, then check against tolerance.

Quantity	How to find it	Value
Absolute error	|98 − 100|	2 cm
Relative error	2 ÷ 100	0.02
Percentage error	0.02 × 100%	2%

Error for a plank of actual length 100 cm measured as 98 cm

Mistakes to expect

Learners often think	Put it right
Significant figures are counted from the first digit written, including zeros at the start.	Count from the first non-zero digit. In 0.005709 the first significant figure is 5, so it has 4 significant figures.
An estimate must be exact.	An estimate uses rounded numbers to show the size of the exact answer, not its exact value.
A measurement with a large absolute error is always unacceptable.	It is acceptable when its absolute error is within the tolerance allowed, so compare the error with the tolerance.

Differentiation

Learners who struggle

Give learners a card with the rounding rule and the steps for standard form. Start with whole numbers that round easily, such as 31 to 30 and 39 to 40.

Learners who finish early

Ask learners who finish early to write the distance from the earth to the moon and a very small length in standard form and round each to 2 significant figures.

Hearing or visual impairment

For learners with hearing impairment, write each rule and each worked line on the board. For learners with visual impairment, read numbers aloud digit by digit, and let them measure a real object by touch and say the value.

No projector, no printing

Use a ruler or string for measuring and a calculator only for checking. All other work is on the board and in exercise books.

If time is short

Teach estimating by rounding and error with tolerance. Leave standard form and significant figures for the next lesson.

Evaluation

Question or task	A correct answer shows
A teacher gives one test a week to a class of 31 pupils and wants a quick estimate of the tests marked in 39 weeks. (a) Round the numbers. (b) Find the estimate. (c) Say what the answer shows.	(a) 31 to 30 and 39 to 40. (b) 30 × 40 = 1,200 tests. (c) It shows the size of the exact answer, about 1,200 tests.
A carpenter cuts a plank that should be 100 cm long but it measures 98 cm, and the customer allows a tolerance of ±1 cm. (a) Find the absolute error. (b) Find the relative error and the percentage error. (c) Say whether the plank is acceptable.	(a) |98 − 100| = 2 cm. (b) 2 ÷ 100 = 0.02 and 0.02 × 100% = 2%. (c) No, 2 cm is more than the tolerance of 1 cm.
A scientist records the distance from the earth to the moon as 384,000,000 metres and a virus length of 2.3 × 10^-8 metres. (a) Write the distance in standard form. (b) Round it to 2 significant figures. (c) Say whether the virus is smaller than one of length 5.1 × 10^-7 metres and give the reason.	(a) 3.84 × 10^8 m. (b) 3.8 × 10^8 m. (c) Yes, because 10^-8 is smaller than 10^-7.

Homework

A farmer has an orchard of 41 trees and expects about 28 mangoes from each tree in a season. (a) Round the numbers to estimate the harvest. (b) Work out the exact total. (c) Find the absolute error of the estimate and say whether it is within a tolerance of 50 mangoes.

Extra practice with answers

Question	Answer
1. A scale shows 698 grams, and a learner is asked to give an answer to 1 significant figure. (a) Say which digit decides the rounding. (b) Round 698 to 1 significant figure. (c) Round 8,600 to 1 significant figure.	(a) The digit after the one kept, 9. (b) 700. (c) 9,000.
2. A learner measures the width of a classroom as 3.746 m. (a) Round to 2 decimal places. (b) Round to 2 significant figures. (c) Say which is more accurate and why.	(a) 3.75 m. (b) 3.7 m. (c) 3.75 m, because it keeps more digits.

3. A parcel of flour has an actual mass of 250 g but the scale reads 245 g, and the shop allows a tolerance of ±10 g. (a) Find the absolute error. (b) Find the percentage error. (c) Say whether it is acceptable.	(a) 5 g. (b) 5 ÷ 250 = 0.02, so 2%. (c) Yes, 5 g is within 10 g.
4. A scientist writes the number 0.04067 and the number 4,560,000 for a report. (a) Round 0.04067 to 2 significant figures. (b) Write 4,560,000 in standard form. (c) Write 0.00072 in standard form.	(a) 0.041. (b) 4.56 × 10^6. (c) 7.2 × 10^-4.
5. A farmer estimates 1,200 mangoes from 41 trees with 28 mangoes each, and her friend says the estimate is too far from the exact value. (a) Find the exact total. (b) Find the absolute error. (c) Say whether it is within a tolerance of 50 mangoes.	(a) 41 × 28 = 1,148. (b) 1,200 − 1,148 = 52 mangoes. (c) No, 52 is more than 50.
6. A trader needs to know quickly the cost of 19 bags of maize at K48 each. (a) Round the numbers. (b) Estimate the cost. (c) Find the exact cost and the absolute error.	(a) 19 to 20 and 48 to 50. (b) 20 × 50 = K1,000. (c) 19 × 48 = K912, so the absolute error is K88.

Answers to the learners' check-yourself questions

1. How many significant figures does 0.005709 have?
   4 (the digits 5, 7, 0 and 9).
2. Write 698.3 to 2 significant figures.
   700.
3. Write 384,000,000 in standard form.
   3.84 × 10^8.
4. A bag holds 9 biscuits. Estimate by rounding the number of biscuits in 41 bags.
   Round 9 to 10 and 41 to 40: 10 × 40 = 400, so about 400 biscuits.
5. What is the absolute error of a measurement of 98 cm when the actual length is 100 cm?
   |98 − 100| = 2 cm.
6. A farmer has an orchard of 41 trees and expects about 28 mangoes from each tree in a season. (a) Round the numbers to estimate the harvest. (b) Work out the exact total on a calculator. (c) Find the absolute error of your estimate and say whether it is within a tolerance of 50 mangoes.
   (a) 40 × 30 = 1,200 mangoes. (b) 41 × 28 = 1,148 mangoes. (c) Absolute error = 1,200 − 1,148 = 52 mangoes, which is more than 50, so the estimate is not within the tolerance.
7. A carpenter cuts a plank that should be 100 cm long but it measures 98 cm. The customer allows a tolerance of ±1 cm. (a) Find the absolute error. (b) Find the percentage error. (c) Is the plank acceptable? Explain your answer.
   (a) 2 cm. (b) (2 ÷ 100) × 100 = 2%. (c) No. The error of 2 cm is greater than the tolerance of 1 cm, so the plank is not acceptable.

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: