Functions and relations

Lesson plan and teacher's notes · Form 1 Mathematics · 1 period of 40 minutes

School	(write school name)	**Teacher**	(write name)
Date	(date)	**Class**	Form 1
Subject	Mathematics	**Enrolment**	Boys ___ Girls ___ Total ___
Time	40 minutes	**Topic**	Functions and relations

This lesson introduces relations as ordered pairs and shows learners how to tell a function from a relation that is not a function. It uses the school clubs and the bus fare from the learners' notes. It comes early in the Form 1 algebra work, before graphs of straight lines.

Specific outcomes

By the end of the lesson, learners should be able to:

1. Write a relation as a set of ordered pairs
2. Decide from a table or mapping diagram whether a relation is a function
3. State the domain and range of a function
4. Use the rule 2n + 3 to find a bus fare and the distance for a given fare

Prior knowledge

- Substituting a number into an expression such as 2n + 3
- Writing a set with curly brackets
- Reading a simple table of values

Teaching and learning materials

- chalk and board
- exercise books
- the learners' note on Functions and relations, if copies are available
- a ruler for the mapping diagram

References

Ministry of Education (CDC) Form 1 Mathematics syllabus and module; 25Schools learners' note: Functions and relations.

Lesson development

Stage and time	Teacher activities	Learner activities
Introduction 5 min	Ask: if a minibus conductor charges K5 for the first kilometre and K2 for each extra kilometre, can two people travelling 4 km pay different fares? Take two or three answers and write on the board: same distance, same fare. Say that today's lesson is about this kind of link.	Answer in pairs, then share with the class. Say why the fare for one distance cannot change.
Development: relations 10 min	Write the club list on the board: Mainza joins Drama, Mazuba joins Music, Chanda joins Debate, Nshimbi joins Debate. Show how to write each link as an ordered pair, learner first. Write the set of four pairs and stress that the order matters.	Copy the ordered pairs. Write one pair for themselves, for example their own name and a favourite subject, and read it aloud.
Development: functions 10 min	Draw the mapping diagram from the board plan. Ask learners to count the arrows leaving each learner. Explain that a function has exactly one arrow from every input, and that two inputs may share one output. Then turn the relation round with clubs as inputs and ask what happens at Debate.	Count the arrows, say whether it is a function and why. Name the type: many-to-one, one-to-one or one-to-many.
Practice 10 min	Give the bus fare rule 2n + 3. Work out the fare for 1 km together, then let learners find the fares for 2 to 5 km and write the domain and range. Walk round and check the substitution. Finish with the question: what distance costs K19?	Complete the table of fares, write the domain {1, 2, 3, 4, 5} and the range {5, 7, 9, 11, 13}, and solve 2n + 3 = 19.
Conclusion 5 min	Ask three learners to say in their own words what a function is. Write the summary: a function gives exactly one output for each input; domain is the set of inputs; range is the set of outputs. Set the homework.	Say what a function is, copy the summary and write down the homework.

Teacher's content notes

Read these before the lesson. They are what you need to know, not what to copy onto the board.

Relation

A relation links members of one set to members of another. Write each link as an ordered pair, input first: (Mainza, Drama). The order matters, because (Drama, Mainza) has a club as the input.

Function

A function is a relation in which every input has exactly one output. Count the arrows leaving each input in a mapping diagram: one arrow each means a function. Two inputs may share an output (Chanda and Nshimbi both join Debate); this is many-to-one and is still a function. One input with two outputs is one-to-many and is not a function.

Domain and range

The domain is the set of all inputs and the range is the set of all outputs. For the bus fare the domain is {1, 2, 3, 4, 5} km and the range is {5, 7, 9, 11, 13} kwacha. Use the words inputs and outputs as often as domain and range.

Where the rule 2n + 3 comes from

The fare is K5 for the first kilometre plus K2 for each of the (n - 1) extra kilometres: 5 + 2(n - 1) = 5 + 2n - 2 = 2n + 3. Check with n = 1: 2 + 3 = 5. For K19, 2n + 3 = 19, so 2n = 16 and n = 8.

Function notation

f(n) = 2n + 3 is read f of n equals two n plus three. f(4) means the output when the input is 4, so f(4) = 2 x 4 + 3 = 11.

How to explain the main idea

Start from something learners know: a fare, a price, a bundle. Ask what would be unfair or confusing if the same input gave two different outputs. Only then give the rule of exactly one output for each input. Learners remember the reason better than the definition.

Questions to ask during the lesson

Question	Expected answer
At a school in Feira, Mainza joins Drama, Mazuba joins Music, Chanda joins Debate and Nshimbi also joins Debate. (a) Write the relation as ordered pairs. (b) Does any learner have two arrows? (c) Is it a function?	(a) {(Mainza, Drama), (Mazuba, Music), (Chanda, Debate), (Nshimbi, Debate)}. (b) No. (c) Yes, many-to-one: every learner has exactly one club.
The head teacher lists the clubs with their members: Debate has Chanda and Nshimbi, and Drama has Mainza. (a) Write the pairs with the club first. (b) How many outputs does Debate have? (c) Is this a function? Why?	(a) {(Debate, Chanda), (Debate, Nshimbi), (Drama, Mainza)}. (b) Two. (c) No: one input has two outputs, so it is one-to-many.
A minibus conductor charges K5 for the first kilometre and K2 for each extra kilometre. (a) Find the fare for 3 km. (b) Write the rule for n km. (c) Name the inputs and the outputs.	(a) 5 + 2 + 2 = K9. (b) 2n + 3. (c) Inputs are distances in km; outputs are fares in kwacha.
A passenger pays K19 on the same minibus, and the conductor says the trip was 8 km. (a) Use 2n + 3 = 19 to find the distance. (b) Show that 8 km gives K19. (c) What does this tell you about the relation?	(a) 2n = 16, so n = 8 km. (b) 2 x 8 + 3 = 19. (c) It is a function: one distance gives one fare.

Worked examples for the board

Example 1: is it a function?

A teacher records the marks of three learners as the relation {(2, 4), (3, 6), (4, 8)}. Each input 2, 3 and 4 appears once, so it is a function. Domain {2, 3, 4}; range {4, 6, 8}. The rule is output = 2 x input.

Example 2: data bundle

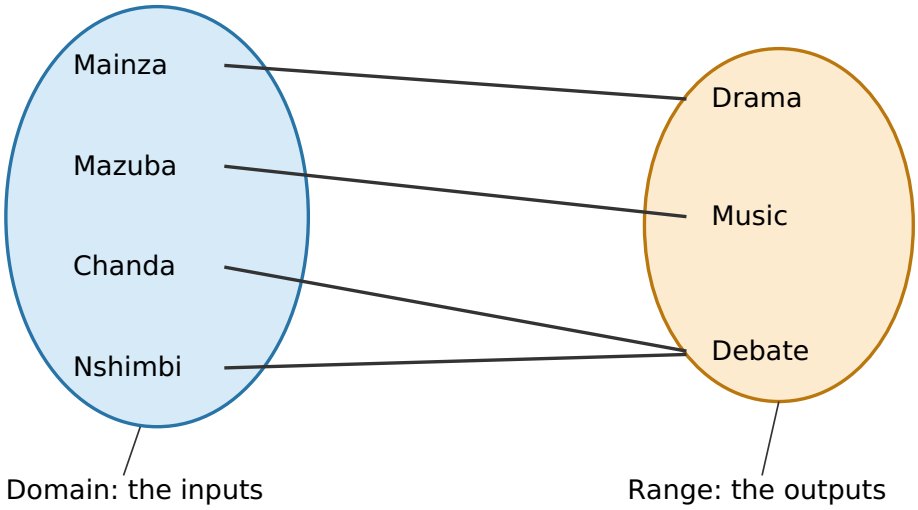
Ngandwe pays K15 for 2 GB and K7 for each extra GB, so C = 7g + 1 for g of at least 2. For 3 GB: 7 x 3 + 1 = K22. For K50: 7g + 1 = 50, 7g = 49, g = 7 GB.

Example 3: temperature

T(h) = 15 + 2h. At 4 hours after 6 AM, T(4) = 15 + 8 = 23 degrees Celsius. The 15 is the starting temperature and the 2 is the rise each hour.

Board plan

Write the title and the date. Left side: the four club pairs as a set, then the sentence a function gives exactly one output for each input. Middle: the mapping diagram, drawn with two ovals and four arrows, with the labels domain and range. Right side: the bus rule 2n + 3 and a table with distances 1 to 5 km. Keep the summary in the bottom corner until the end.



The mapping diagram the teacher draws on the board: four learners, three clubs, one arrow from each learner

Mistakes to expect

Learners often think	Put it right
Two inputs with the same output means the relation is not a function.	Show Chanda and Nshimbi both joining Debate. Each learner still has one arrow, so it is a function. Only an input with two arrows breaks the rule.

The order of an ordered pair does not matter.	Write (Mainza, Drama) and (Drama, Mainza) and ask what each says. One is a learner joined to a club; the other has a club as the input.
The domain is the answers and the range is the questions.	Use the words inputs and outputs every time. Domain is the distances, range is the fares.

Differentiation

Learners who struggle

Give struggling learners the ordered pairs already written and ask only whether any name appears twice on the left. For the bus rule, let them build the fares one at a time by adding K2 to the last fare before using the rule.

Learners who finish early

Ask learners who finish early to write the rule for a taxi that charges K10 for the first kilometre and K3 for each extra kilometre, to make a table for 1 to 5 km and to say how far a K25 fare takes a passenger.

Hearing or visual impairment

For learners with hearing impairment, face the class when speaking, write each instruction on the board and use the arrows and ovals to show links. For learners with visual impairment, seat them near the board, read out every pair and arrow aloud, and let them build the relation with named cards or objects they can touch.

No projector, no printing

Everything is taught with chalk and the board. Draw the mapping diagram once and let learners copy it into their books. Use learners' own names for the first relation, so no printed list is needed.

If time is short

Keep the club relation and the mapping diagram, which carry the main idea. Move the bus-fare table to homework and do the rule 2n + 3 in the next lesson.

Evaluation

Question or task	A correct answer shows
ZAME gives each member an identity number: 1001 Chimfwembe, 1002 Mangwatu and 1003 Sibeso. (a) Write the ordered pairs. (b) State the domain and range. (c) Is it a function? Give a reason.	(a) {(1001, Chimfwembe), (1002, Mangwatu), (1003, Sibeso)}. (b) Domain {1001, 1002, 1003}; range the three names. (c) Yes, each number has one name.
A fruit vendor in Soweto market sells oranges at K2 each. (a) Write the rule for her income I from x oranges. (b) Find the income for 5 and 20 oranges. (c) State the domain and range for 5, 10, 15 and 20 oranges.	(a) I = 2x. (b) K10 and K40. (c) Domain {5, 10, 15, 20}; range {10, 20, 30, 40}.

A car on the Great East Road uses 1 litre of fuel for every 12 km. (a) Write the fuel L for d km. (b) Find the fuel used for 60 km. (c) How far does 3 litres take the car?	(a) L = d / 12. (b) 5 litres. (c) 36 km.

Homework

A fruit vendor sells oranges at K2 each and wants to know her income. (a) Write the rule for her income from x oranges. (b) Find the income for 5, 10, 15 and 20 oranges. (c) Write the domain and range.

Extra practice with answers

Question	Answer
1. A taxi in Lusaka charges K10 for the first kilometre and K3 for each extra kilometre. (a) Write the fare for n km. (b) Find the fare for 5 km. (c) How far can a passenger travel for K25?	(a) 10 + 3(n - 1) = 3n + 7. (b) K22. (c) 3n + 7 = 25, so n = 6 km.
2. Ngandwe pays K15 for a 2 GB data bundle and K7 for each extra GB, so C = 7g + 1 for g of at least 2. (a) Find the cost of 5 GB. (b) How many GB can she buy for K50? (c) Write the pairs for g = 2, 3, 4.	(a) K36. (b) 7g = 49, g = 7 GB. (c) (2, 15), (3, 22), (4, 29).
3. At 6 AM the temperature in the village is 15 degrees Celsius and it rises by 2 degrees every hour. (a) Write the rule T(h). (b) Find T at 9 AM. (c) At what time is it 23 degrees?	(a) T(h) = 15 + 2h. (b) h = 3, so 21 degrees. (c) 15 + 2h = 23, h = 4, so 10 AM.
4. At a sports day Joe plays football, Tina plays netball and Bwalya plays football. (a) Write the ordered pairs. (b) State the domain and range. (c) Is it a function? Why?	(a) {(Joe, Football), (Tina, Netball), (Bwalya, Football)}. (b) Domain {Joe, Tina, Bwalya}; range {Football, Netball}. (c) Yes, each learner has one sport.
5. A tailor at the market sews dresses at K20 each, and her takings depend on the number of dresses sewn. (a) Write the rule for takings T from d dresses. (b) Find the takings for 7 dresses. (c) How many dresses give K260?	(a) T = 20d. (b) K140. (c) 20d = 260, so d = 13 dresses.
6. A learner records a rule in a table: input 1 gives 4, input 2 gives 7 and input 3 gives 10, and she thinks the rule is y = 3x + 1. (a) Check the rule for input 2. (b) Find y when x = 5. (c) State the range of the table.	(a) 3 x 2 + 1 = 7, correct. (b) 16. (c) {4, 7, 10}.

Answers to the learners' check-yourself questions

1. Is {(1001, Chimfwembe), (1002, Mangwatu), (1003, Sibeso)} a function? Give a reason.

Yes. Each identity number is linked to only one name, so every input has exactly one output.

2. State the domain and range of {(1, 3), (2, 5), (3, 7)}.
   Domain {1, 2, 3}; range {3, 5, 7}.
3. Is the relation {(Debate, Chanda), (Debate, Nshimbi), (Drama, Mainza)} a function? Explain.
   No. The input Debate has two outputs, Chanda and Nshimbi, so it is one-to-many.
4. If f(x) = 3x - 2, find f(4).
   f(4) = 3 x 4 - 2 = 12 - 2 = 10.
5. A car uses 1 litre of fuel for every 12 km. The fuel used is L = d / 12, where d is the distance in km. (a) How much fuel is used for 60 km? (b) How far can the car go on 3 litres? (c) Is this relation a function of d? Explain.
   (a) L = 60 / 12 = 5 litres. (b) d / 12 = 3, so d = 36 km. (c) Yes, because each distance gives exactly one amount of fuel.
6. Ngandwe pays K15 for a 2 GB data bundle and K7 for every extra GB, so the cost for g GB (g at least 2) is C = 7g + 1. (a) Find the cost of 5 GB. (b) How many GB can she buy for K50? (c) State the first three inputs and outputs for g = 2, 3, 4.
   (a) C = 7 x 5 + 1 = K36. (b) 7g + 1 = 50, so 7g = 49 and g = 7 GB. (c) (2, 15), (3, 22), (4, 29).

Teacher's reflection

What went well:

What I will change next time:

Learners who need follow-up: