25SCHOOLS

FREE LEARNING NOTES FOR ZAMBIA

FORM 1 · TERM 1

Mathematics

Functions and relations

Lesson notes for learners · Competence-Based Curriculum

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A taxi conductor charges K5 for the first kilometre and K2 for every extra kilometre. Once you know how far you travel, the fare is fixed: nobody can pay two different fares for the same journey. This special kind of link between two sets of numbers is called a function. In this note you learn what a relation is, when a relation is a function, and how to describe one with a table, a rule and a mapping diagram.

AFTER THESE NOTES YOU SHOULD BE ABLE TO

- Write a relation as ordered pairs and show it in a mapping diagram
- Decide whether a relation is a function
- State the domain and range of a function
- Use a rule and function notation to find outputs and inputs

KEY WORDS

relation	a link between the members of one set and the members of another set
ordered pair	two values written in brackets in a fixed order, such as (Mainza, Drama)
function	a relation in which every input has exactly one output
domain	the set of all inputs of a function
range	the set of all outputs of a function
mapping diagram	a picture that uses arrows to join each input to its output

1 Relations and ordered pairs

A relation links things from one set to things in another set. In a school, each learner joins a club. The learners form the first set and the clubs form the second set. We write each link as an ordered pair, with the learner first and the club second.

Suppose Mainza joins Drama, Mazuba joins Music, Chanda joins Debate and Nshimbi joins Debate. The relation is {(Mainza, Drama), (Mazuba, Music), (Chanda, Debate), (Nshimbi, Debate)}. The order matters: (Mainza, Drama) is a learner joined to a club, not a club joined to a learner.

Learner (input)	Club (output)	Ordered pair
Mainza	Drama	(Mainza, Drama)
Mazuba	Music	(Mazuba, Music)
Chanda	Debate	(Chanda, Debate)
Nshimbi	Debate	(Nshimbi, Debate)

Figure 1. The learner and club relation written as ordered pairs

2 When a relation is a function

A function is a relation in which every input has exactly one output. In the club relation each learner joins only one club, so it is a function. Two learners, Chanda and Nshimbi, share the output Debate, and that is allowed. This type is called many-to-one.

Now turn the relation around so that clubs are the inputs. Debate would be linked to both Chanda and Nshimbi. One input with two outputs is called one-to-many, and it is not a function. A one-to-one relation, such as a list of members of ZAME with a different identity number each, is a function too.

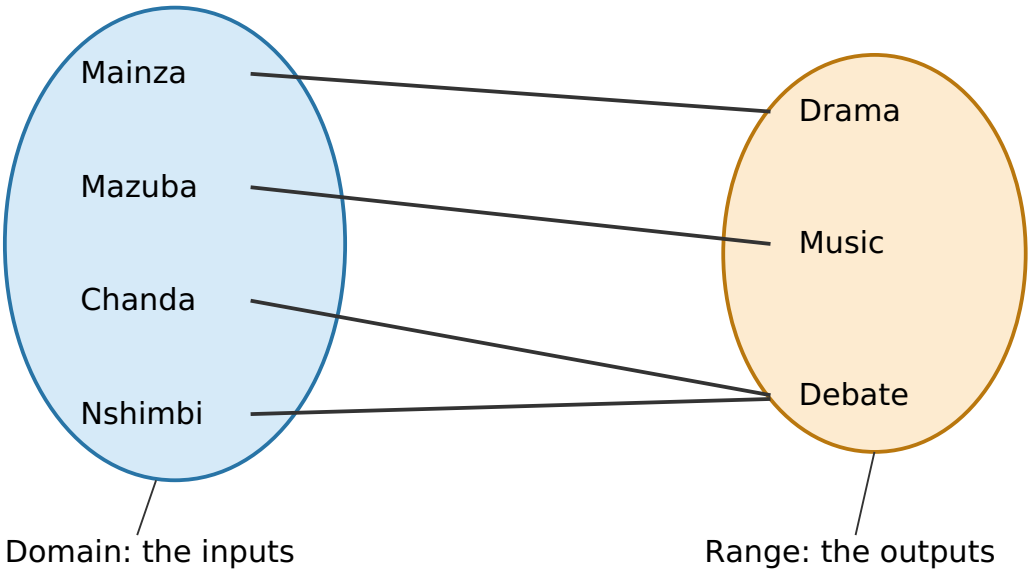


Figure 2. A mapping diagram: every learner has exactly one arrow, so the relation is a function

3 Rules, domain and range

Many functions follow a rule. A bus charges K5 for the first kilometre and K2 for each extra kilometre. For a journey of n kilometres the fare is 5 + 2 x (n - 1), which simplifies to 2n + 3. The inputs are the distances 1, 2, 3, 4 and 5 km, so the domain is {1, 2, 3, 4, 5}. The outputs are the fares, so the range is {5, 7, 9, 11, 13}.

A fruit vendor in Soweto market sells oranges at K2 each. Her income is 2 times the number of oranges sold. For 5, 10, 15 and 20 oranges the incomes are K10, K20, K30 and K40. The domain is {5, 10, 15, 20} and the range is {10, 20, 30, 40}.

WORKED EXAMPLE

Using the bus rule, find the fare for 8 km, and the distance travelled when the fare is K19.

Fare = 2n + 3

For 8 km: 2 x 8 + 3 = 16 + 3 = 19, so the fare is K19

For K19: 2n + 3 = 19

2n = 16, so n = 8

The journey is 8 km

4 Function notation and graphs

We name a function with a letter such as f and write f(x) for the output when the input is x. The bus rule becomes f(n) = 2n + 3, so f(4) = 2 x 4 + 3 = 11. This means a 4 km journey costs K11.

At 6 AM the temperature is 15 degrees Celsius and it rises by 2 degrees every hour. After h hours the temperature is T(h) = 15 + 2h. Plot the ordered pairs (0, 15), (1, 17), (2, 19) and (3, 21), and the points lie on a straight line. The 15 is where the line starts and the 2 tells you how steeply it rises.

Hours after 6 AM (h)	Temperature T(h) in degrees Celsius
0	15
1	17
2	19
3	21
4	23

Figure 3. Temperature after 6 AM using T(h) = 15 + 2h

TRY THIS

Think of something in your home that depends on one number, for example the cost of buying airtime or the water you use in buckets. Write a rule for it, make a table with four inputs, write the ordered pairs, state the domain and range, and plot the points on graph paper.

REMEMBER

- A relation links members of one set to members of another and is written as ordered pairs.
- A function is a relation in which every input has exactly one output.
- Many-to-one and one-to-one relations are functions; one-to-many is not.
- The domain is the set of inputs and the range is the set of outputs.
- f(x) means the output of the function f when the input is x.

? Check yourself

1. Is {(1001, Chimfwembe), (1002, Mangwatu), (1003, Sibeso)} a function? Give a reason.
2. State the domain and range of {(1, 3), (2, 5), (3, 7)}.
3. Is the relation {(Debate, Chanda), (Debate, Nshimbi), (Drama, Mainza)} a function? Explain.
4. If f(x) = 3x - 2, find f(4).
5. A car uses 1 litre of fuel for every 12 km. The fuel used is L = d / 12, where d is the distance in km. (a) How much fuel is used for 60 km? (b) How far can the car go on 3 litres? (c) Is this relation a function of d? Explain.
6. Ngandwe pays K15 for a 2 GB data bundle and K7 for every extra GB, so the cost for g GB (g at least 2) is C = 7g + 1. (a) Find the cost of 5 GB. (b) How many GB can she buy for K50? (c) State the first three inputs and outputs for g = 2, 3, 4.

ANSWERS

1. Yes. Each identity number is linked to only one name, so every input has exactly one output.
2. Domain {1, 2, 3}; range {3, 5, 7}.
3. No. The input Debate has two outputs, Chanda and Nshimbi, so it is one-to-many.
4. f(4) = 3 x 4 - 2 = 12 - 2 = 10.
5. (a) L = 60 / 12 = 5 litres. (b) d / 12 = 3, so d = 36 km. (c) Yes, because each distance gives exactly one amount of fuel.
6. (a) C = 7 x 5 + 1 = K36. (b) 7g + 1 = 50, so 7g = 49 and g = 7 GB. (c) (2, 15), (3, 22), (4, 29).

Written by 25Schools to follow the Form 1 Mathematics syllabus of the Zambian competence-based curriculum. Free for learners to read, download and share. More notes and quizzes at 25schools.com.