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FREE LEARNING NOTES FOR ZAMBIA

FORM 2

Mathematics

Systems of equations: simultaneous equations

Lesson notes for learners · Competence-Based Curriculum

INSIDE

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Many everyday problems hide two unknown amounts, such as the price of an exercise book and the price of a pen at the school shop. When two conditions must be true at the same time, we write two equations and find the values that fit both. This is how a trader or a learner can work out prices from two receipts.

AFTER THESE NOTES YOU SHOULD BE ABLE TO

- Explain what a variable, an equation and a system of equations are
- Write equations for real situations with one or two unknown quantities
- Solve an equation in one variable using the balance idea and check the answer
- Solve simultaneous equations by elimination and state the answer in the situation

KEY WORDS

variable	a symbol such as x or y that stands for an unknown or changing quantity
equation	a statement that two expressions have the same value
linear equation	an equation such as ax + b = c, or ax + by = c with two variables
system of equations	two or more equations considered together
simultaneous equations	equations with the same unknowns that must both be true at the same time
solution	values of the unknowns that satisfy the equation or equations
elimination	removing one variable by adding or subtracting suitable equations
coefficient	the number that multiplies a variable, such as 2 in 2x

1 Unknowns and equations

When a quantity is not known, we let a variable such as x or y stand for it. An equation says that two expressions have the same value. Some situations need one equation in one variable. Others have two unknown quantities and need two equations that are true at the same time.

Situation A: three identical exercise books and a ruler costing K6 have a total cost of K39. Let x be the cost of one exercise book. One unknown is enough, so one equation can be written: 3x + 6 = 39.

Situation B: at a school shop, 2 exercise books and 1 pen cost K23, and 1 exercise book and 2 pens cost K19. Let x be the price of an exercise book and y the price of a pen. There are two unknowns, so we write two equations: 2x + y = 23 and x + 2y = 19. Two equations that are considered together form a system of equations, and a solution must satisfy both of them at the same time.

One equation is not enough in Situation B. Many pairs of prices fit 2x + y = 23 on its own, but only one pair also fits x + 2y = 19.

	Situation A	Situation B
Unknown quantities	cost of one exercise book (x)	price of a book (x) and of a pen (y)
Equations needed	one: 3x + 6 = 39	two: 2x + y = 23 and x + 2y = 19
Linear equation form	ax + b = c	ax + by = c

Figure 1. One unknown or two unknowns

2 Solving an equation in one variable

An equation behaves like a balance. Whatever you do to one side, you must do to the other side to keep it balanced. Use inverse operations to get the variable on its own, and check your answer by putting it back into the original equation.

Always say what the answer means. If x is the cost of an exercise book, then x = 9 means one exercise book costs K9.

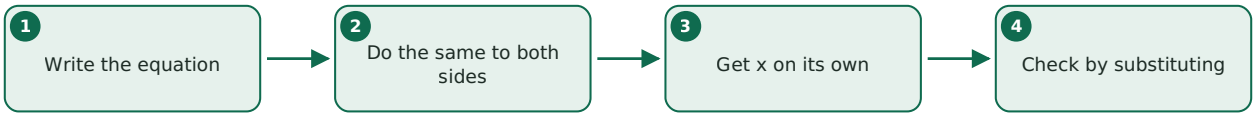

Figure 2. Solving a one-variable equation

WORKED EXAMPLE

Solve 3x - 7 = 20.

3x - 7 = 20

Add 7 to both sides: 3x = 27

Divide both sides by 3: x = 9

Check: 3(9) - 7 = 27 - 7 = 20, so x = 9 is correct

3 Solving simultaneous equations by elimination

Elimination removes one variable by adding or subtracting suitable equations. If the coefficients of a variable are not equal or opposite, multiply one or both equations by suitable numbers first. After you find one variable, substitute its value into either original equation to find the other.

If one variable already has the same coefficient in both equations, you can eliminate it at once. Subtract the equations when the coefficients have the same sign, and add them when the coefficients are opposites.

The module also names substitution and the graphical method as ways of solving simultaneous equations. This note shows elimination.

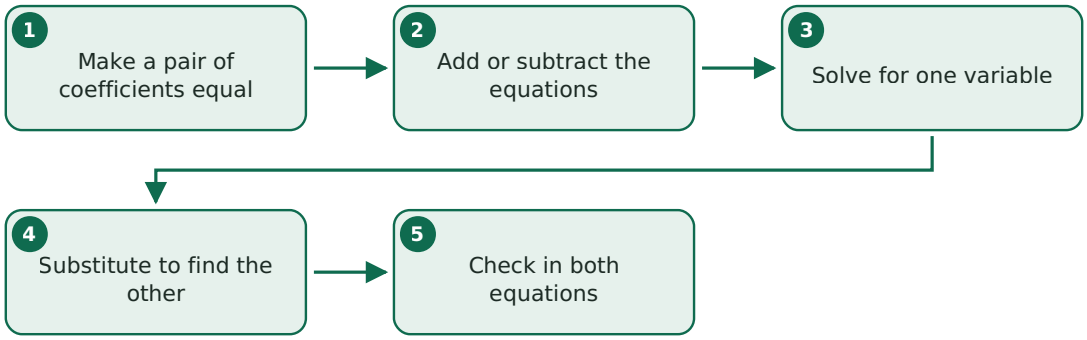

Figure 3. Elimination in five steps

WORKED EXAMPLE

At a market, 2 kg of tomatoes and 3 kg of onions cost K42. 3 kg of tomatoes and 2 kg of onions cost K43. Find the price per kilogram of each. Let x be tomatoes and y be onions.

2x + 3y = 42 (1) and 3x + 2y = 43 (2)

Multiply (1) by 3: 6x + 9y = 126. Multiply (2) by 2: 6x + 4y = 86

Subtract the second from the first: 5y = 40, so y = 8

Substitute y = 8 into (1): 2x + 24 = 42, so x = 9

Tomatoes cost K9 per kilogram and onions cost K8 per kilogram

4 Using simultaneous equations in real life

To solve a real problem, define the variables clearly, write one equation for each condition, solve, check in both equations and then state the answer in words, not only as x and y.

Return to Situation B. The equations are 2x + y = 23 and x + 2y = 19. Multiply the second equation by 2 to get 2x + 4y = 38. Subtract the first equation: 3y = 15, so y = 5. Then 2x + 5 = 23, so x = 9. Check in the second equation: 9 + 2(5) = 19, which is correct. An exercise book costs K9 and a pen costs K5.

Equation	With x = 9 and y = 5	True?
2x + y = 23	2(9) + 5 = 23	yes
x + 2y = 19	9 + 2(5) = 19	yes

Figure 4. Checking the school shop answer

WORKED EXAMPLE

Solve the system 2x + y = 19 and 2x - y = 5.

The 2x terms are equal and y has opposite signs

To eliminate y, add the equations: 4x = 24, so x = 6

Substitute x = 6 into 2x + y = 19: 12 + y = 19, so y = 7

Check in 2x - y = 5: 12 - 7 = 5, which is correct

The solution is x = 6 and y = 7

TRY THIS

With a partner, pick two purchases from a real shop or market that share the same two items, for example 2 of item A and 1 of item B, then 1 of A and 2 of B. Write the totals, then write two equations, solve them by elimination and check the prices against the shop's real prices.

REMEMBER

- A variable stands for an unknown quantity; an equation says two expressions are equal.
- A system of equations has two or more equations that must be true at the same time.
- An equation is a balance: do the same to both sides, then check by substituting.
- Elimination removes one variable by adding or subtracting equations, after multiplying first if needed.
- Find one variable, substitute to find the other, then check in both equations.
- State the final answer in the context of the problem.

? Check yourself

1. What does a variable stand for in an equation?
2. Solve 4x - 3 = 21 and check your answer.
3. Why must the same x and y be used in both equations of a system?
4. Solve x + y = 10 and x - y = 4.
5. You are a learner at a school tuck shop in Chipata. You buy 3 pencils and 1 ruler for K13. Your friend buys 1 pencil and 1 ruler for K7. (a) Write two equations, letting p be the price of a pencil and r the price of a ruler. (b) Solve them by elimination. (c) State the price of each item and check y
6. You sell maize and beans at a market near Feira. One customer pays K30 for 2 kg of maize and 1 kg of beans. Another pays K35 for 1 kg of maize and 3 kg of beans. (a) Write two equations with m for maize and b for beans, both per kilogram. (b) Find the price per kilogram of each. (c) Explain what you

ANSWERS

1. An unknown or changing quantity, shown by a symbol such as x or y.
2. 4x = 24, so x = 6. Check: 4(6) - 3 = 21, which is correct.
3. Both equations describe the same two unknown quantities, and the solution must satisfy both at the same time.
4. Add the equations: 2x = 14, so x = 7. Then 7 + y = 10, so y = 3. Check: 7 - 3 = 4, which is correct.
5. (a) 3p + r = 13 and p + r = 7. (b) Subtract the second from the first: 2p = 6, so p = 3. Then 3 + r = 7, so r = 4. (c) A pencil costs K3 and a ruler costs K4. Check: 3(3) + 4 = 13 and 3 + 4 = 7, both correct.
6. (a) 2m + b = 30 and m + 3b = 35. (b) Multiply the second equation by 2: 2m + 6b = 70. Subtract the first: 5b = 40, so b = 8. Then 2m + 8 = 30, so m = 11. (c) Maize costs K11 per kilogram and beans cost K8 per kilogram. Check: 2(11) + 8 = 30 and 11 + 3(8) = 35.

Written by 25Schools to follow the Form 2 Mathematics syllabus of the Zambian competence-based curriculum. Free for learners to read, download and share. More notes and quizzes at 25schools.com.