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FREE LEARNING NOTES FOR ZAMBIA

FORM 2

Mathematics

Relations: types, arrow diagrams and ordered pairs

Lesson notes for learners · Competence-Based Curriculum

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Money earned depends on hours worked, a learner is linked to an ID number, and a teacher is linked to the learners they teach. Mathematics describes connections like these using relations. In this note you learn what a relation is, the four types of relation, how to show a relation with an arrow diagram and ordered pairs, and how a function fits in.

AFTER THESE NOTES YOU SHOULD BE ABLE TO

- Explain what a relation between two sets is
- Name the four types of relation: one-to-one, one-to-many, many-to-one and many-to-many
- Show a relation using an arrow diagram and ordered pairs
- Find the elements of set B for a given set A and relation
- Say what a function is

KEY WORDS

relation	a connection that describes how elements in one set are matched to elements in another set
ordered pair	a pair of elements written (a, b), where a is from the first set and b is from the second
arrow diagram	a picture, also called a mapping, that uses arrows to join elements of one set to elements of another
input	an element of the first set
output	the element of the second set that an input is linked to
function	a special relation in which each input is linked to exactly one output

1 What is a relation

A relation describes how the elements of one set are connected or matched to the elements of another set. For example, the set of learners and the set of ID numbers are connected by the relation 'is the ID for'. A relation can be shown as ordered pairs, as a table, as an arrow diagram (mapping) or as a graph.

In this note we use ordered pairs and arrow diagrams.

2 The four types of relation

We name a relation by looking at how many items are joined on each side. In a one-to-one relation each item in the first set is linked to only one item in the second set, and each item in the second set is linked to only one item in the first. Learners and their ID numbers are an example.

In a one-to-many relation one item in the first set is linked to more than one item in the second set, while each item in the second set is linked to only one in the first. 'Is the father of' is an example, because one father can have several children.

In a many-to-one relation several items in the first set are linked to the same item in the second set. In a many-to-many relation several items in the first set are linked to several items in the second set. 'Is the teacher of' is an example, because a learner has many teachers and a teacher has many learners.

Type	Meaning	Example
One-to-one	Each item is linked to exactly one item on the other side	Learner and ID number
One-to-many	One item in set A is linked to several items in set B	Father and his children
Many-to-one	Several items in set A are linked to one item in set B	Learners and the class they are in
Many-to-many	Several items in set A are linked to several in set B	Teachers and learners

Figure 1. The four types of relation

3 Arrow diagrams and ordered pairs

An arrow diagram shows each set as a closed shape. An arrow goes from each element of set A to the element of set B that it is linked to. The same relation can be written as ordered pairs, with the element of A first and the element of B second.

To find set B, apply the rule to every element of A.

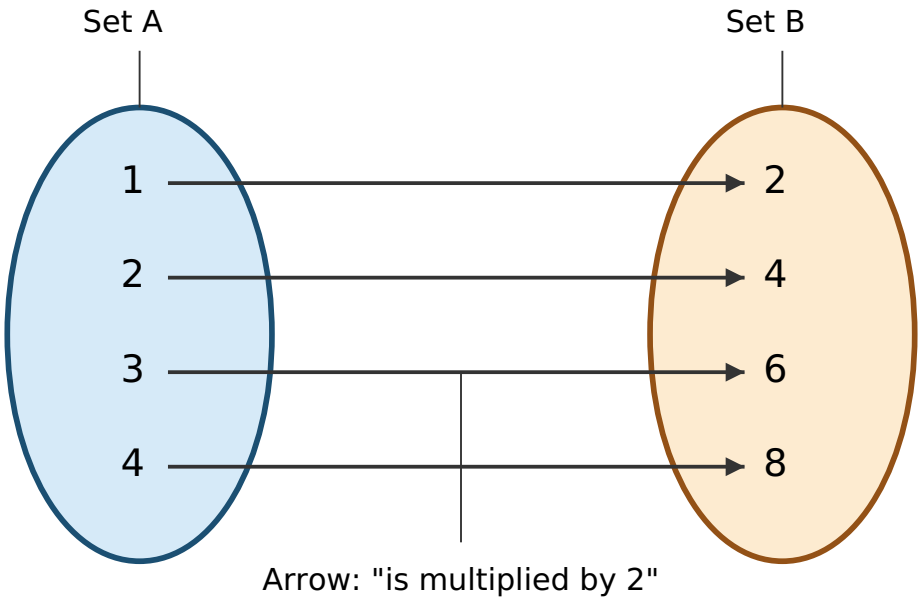

Figure 2. Arrow diagram of the relation 'is multiplied by 2' from set A to set B

WORKED EXAMPLE

Set A = {1, 2, 3, 4} and the relation is 'is multiplied by 2'. Find set B, write the ordered pairs and name the type of relation.

Multiply each element of A by 2: 1 gives 2, 2 gives 4, 3 gives 6, 4 gives 8

Set B = {2, 4, 6, 8}

Ordered pairs: (1, 2), (2, 4), (3, 6), (4, 8)

Each element of A has one arrow and each element of B receives one arrow, so the relation is one-to-one

4 Relations that are functions

A function is a special type of relation in which each input is linked to exactly one output. Look at the arrows from each element of set A. If every element has exactly one arrow leaving it, the relation is a function.

So one-to-one and many-to-one relations are functions, because each input has only one arrow. One-to-many and many-to-many relations are not functions, because some input has more than one arrow.

WORKED EXAMPLE

Set A = {2, 3, 4, 5} and the relation is 'is a square root of'. Find set B and write the ordered pairs. Is this relation a function?

The square root of 4 is 2, of 9 is 3, of 16 is 4 and of 25 is 5, so set B = {4, 9, 16, 25}

Ordered pairs: (2, 4), (3, 9), (4, 16), (5, 25)

Each element of A has exactly one arrow, so it is one-to-one

Because each input has exactly one output, it is a function

TRY THIS

In groups, make four cards, each with two real sets, for example learners and ID numbers, teachers and subjects taught, learners and favourite sports, countries and languages spoken. List at least five elements in each set, draw an arrow diagram for each pair, write some ordered pairs, and name the type of relation. Present your work to the class.

REMEMBER

- A relation describes how elements of one set are matched to elements of another
- The four types are one-to-one, one-to-many, many-to-one and many-to-many
- A relation can be shown as ordered pairs or as an arrow diagram
- A function is a relation in which each input has exactly one output
- One-to-one and many-to-one relations are functions; one-to-many and many-to-many are not

? Check yourself

1. What is a relation between two sets?
2. Name the four types of relation.
3. Write the relation 'add 3' from A = {1, 2, 3} as ordered pairs.
4. Is a one-to-many relation a function? Explain.
5. What type of relation is 'is the ID for' between learners and ID numbers?
6. You are a learner at a school in Luangwa. Four learners, Mulenga, Thokozile, Chileshe and Nandi, are each linked to the club they belong to: Mulenga and Chileshe belong to

the Science Club, while Thokozile and Nandi belong to the Drama Club. (a) Draw in your book an arrow diagram for the relation 'b

7. A fish seller at a market in Feira sells bream and tilapia, and each buyer, Mr Banda, Mrs Zulu and Mr Phiri, buys from one or both. Mr Banda buys bream and tilapia, Mrs Zulu buys bream, and Mr Phiri buys tilapia. (a) Write the ordered pairs for 'buys'. (b) State the type of relation. (c) Explain why

ANSWERS

1. A connection that describes how the elements of one set are matched to the elements of another set.
2. One-to-one, one-to-many, many-to-one and many-to-many.
3. (1, 4), (2, 5), (3, 6).
4. No, because some input is linked to more than one output.
5. One-to-one.
6. (a) Arrows from Mulenga and Chileshe to Science Club, and from Thokozile and Nandi to Drama Club. (b) (Mulenga, Science), (Chileshe, Science), (Thokozile, Drama), (Nandi, Drama). (c) Many-to-one; it is a function because each learner has exactly one club.
7. (a) (Banda, bream), (Banda, tilapia), (Zulu, bream), (Phiri, tilapia). (b) Many-to-many. (c) Mr Banda is linked to two items, so one input has more than one output.

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