25SCHOOLS

FREE LEARNING NOTES FOR ZAMBIA

FORM 1

Mathematics

Approximations and estimation

Lesson notes for learners · Competence-Based Curriculum

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We often need a quick answer that is close enough, such as how many exercise books to buy for a class or how much each learner must pay for a trip. Estimation and approximation let us do this by rounding numbers and by knowing how far an answer may be from the true value. This note covers the main ideas of the topic.

AFTER THESE NOTES YOU SHOULD BE ABLE TO

- Round numbers to estimate an answer without a calculator
- Round to a given number of significant figures or decimal places
- Write a large or small number in standard form
- Find absolute, relative and percentage error and compare it with a tolerance

KEY WORDS

estimate	a rough answer found by using rounded numbers
approximate	close to the exact value but not exactly equal to it
significant figures	the digits of a number that carry its value, counted from the first non-zero digit
standard form	a number written as a number between 1 and 10 multiplied by a power of 10
absolute error	the size of the difference between the measured value and the actual value
relative error	the absolute error divided by the actual value
percentage error	the relative error written as a percentage
tolerance	the largest error that is allowed

1 Estimating in everyday life

Estimation is used in daily activities, such as planning a class trip, counting a harvest or finding how many tests a teacher will mark. To estimate, round each number to a simple value, then calculate with the rounded values. The answer is not exact but it shows the size of the exact answer.

A teacher gives one test a week to a class of 31 pupils. To estimate the number of tests marked in 39 weeks, round 31 to 30 and 39 to 40. Then $30 \times 40 = 1{,}200$, so about 1,200 tests are marked.

Rounding can be to the nearest whole unit, fraction or decimal, depending on how close the answer must be.

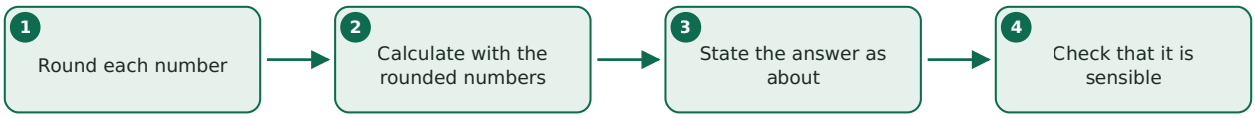

Figure 1. How to estimate

WORKED EXAMPLE

A class trip has 15 learners. The bus costs K485, lunch is K95 per learner and the park entrance is about K175 per learner. Estimate how much each learner pays, without a calculator.

Round K485 to K500, K95 to K100 and K175 to K200

Cost per learner for lunch and entrance = 100 + 200 = K300

Total for 15 learners = 15 × 300 = K4,500, and with the bus 4,500 + 500 = K5,000

Each learner pays about 5,000 ÷ 15, which is about K330

2 Rounding and significant figures

Measurements are approximated by rounding to a chosen degree of accuracy. You may round to the nearest unit, fraction or decimal place, or to a number of significant figures.

To count significant figures, start at the first non-zero digit and count from there. In 0.005709 the first significant figure is 5, so the number has 4 significant figures.

To round to a number of significant figures, look at the digit after the last one you keep. If it is 5 or more, increase the last kept digit by 1. If it is less than 5, leave it. Fill the remaining places up to the decimal point with zeros.

Number	Rounded to	Answer
698.3	2 significant figures	700
0.00107645	3 significant figures	0.00108
0.005709	2 decimal places	0.01

Figure 2. Rounding examples

3 Standard form

Very large and very small numbers are written in standard form, also called scientific notation. A number in standard form is written as a × 10 to a power, where a is between 1 and 10.

The distance from the earth to the moon is 384,000,000 metres. In standard form this is 3.84 × 10^8 metres. Rounded to 2 significant figures it is 3.8 × 10^8 metres.

A virus that measures about 2.3 × 10^-8 metres is smaller than one that measures 5.1 × 10^-7 metres, because 10^-7 is larger than 10^-8. Numbers in standard form can also be rounded to a given degree of accuracy.

4 Error and tolerance

No measurement is exact, so we describe how far it is from the actual value. The absolute error is the difference between the measured and actual values, taken as a positive number. The relative error is the absolute error divided by the actual value. Multiply it by 100 to get the percentage error.

A tolerance is the error that is allowed. A measurement is acceptable when its absolute error is within the tolerance.

Quantity	Working	Result
Absolute error	100 − 98	2 cm
Relative error	2 ÷ 100	0.02
Percentage error	0.02 × 100	2%
Tolerance ±3 cm	2 is less than 3	Acceptable

Figure 3. A carpenter measures wood of actual length 100 cm and gets 98 cm

TRY THIS

In groups, plan a school sports day for 40 learners. Use rounded numbers, without a calculator, to estimate the cost of water at K6 per bottle and transport at K390. Then work out the exact cost on a calculator, find the absolute error and percentage error of your estimate and say whether it is within a tolerance of K50.

REMEMBER

- An estimate is found by rounding numbers and calculating with them.
- Numbers can be rounded to the nearest unit, fraction, decimal place or significant figures.
- Standard form is a × 10 to a power, with a between 1 and 10.
- Absolute error is the difference between measured and actual value.
- Relative error is the absolute error divided by the actual value; times 100 it is the percentage error.
- A measurement is acceptable if its error is within the tolerance.

? Check yourself

1. How many significant figures does 0.005709 have?
2. Write 698.3 to 2 significant figures.
3. Write 384,000,000 in standard form.
4. A bag holds 9 biscuits. Estimate by rounding the number of biscuits in 41 bags.
5. What is the absolute error of a measurement of 98 cm when the actual length is 100 cm?
6. A farmer has an orchard of 41 trees and expects about 28 mangoes from each tree in a season. (a) Round the numbers to estimate the harvest. (b) Work out the exact total on a calculator. (c) Find the absolute error of your estimate and say whether it is within a tolerance of 50 mangoes.
7. A carpenter cuts a plank that should be 100 cm long but it measures 98 cm. The customer allows a tolerance of ±1 cm. (a) Find the absolute error. (b) Find the percentage error. (c) Is the plank acceptable? Explain your answer.

ANSWERS

1. 4 (the digits 5, 7, 0 and 9).
2. 700.
3. 3.84 × 10^8.
4. Round 9 to 10 and 41 to 40: 10 × 40 = 400, so about 400 biscuits.
5. |98 − 100| = 2 cm.
6. (a) 40 × 30 = 1,200 mangoes. (b) 41 × 28 = 1,148 mangoes. (c) Absolute error = 1,200 − 1,148 = 52 mangoes, which is more than 50, so the estimate is not within the tolerance.
7. (a) 2 cm. (b) (2 ÷ 100) × 100 = 2%. (c) No. The error of 2 cm is greater than the tolerance of 1 cm, so the plank is not acceptable.

Written by 25Schools to follow the Form 1 Mathematics syllabus of the Zambian competence-based curriculum. Free for learners to read, download and share. More notes and quizzes at 25schools.com.